1. The z-axis coincides with the axis of an infinite conducting cylinder of inner radius $a$, outer radius $b$. For $a < r < b$, there is a current density flowing around the wire, $\vec{J} = J_0 \hat{\phi}$ A/m$^2$.

For the three regions

(I) $0 < r < a$,

(II) $a < r < b$

(III) $b < r$,

Find:

(a) The $\mathbf{B}$ field.

(b) The $\mathbf{A}$ field.

And then find

(c) The pressure exerted by the magnetic field on the cylinder.
2. A grounded conducting sphere of radius $a$ is inside a sphere of radius $b$. The region between radius $a$ and radius $b$ is vacuum. The outer sphere is non-conducting, and carries a surface charge density $\sigma_0 \cos \theta$. In the regions:

(I) $a < r < b$

(II) $b < r$

Find the potential $\Phi$. 
A hollow grounded conducting sphere of radius $R$ contains a point charge $q$ at the point $\mathbf{a}$. 

(a) Find the potential inside the sphere.

(b) Find the vector force on the charge $q$.

The sphere of the previous problem is now reduced to a conducting hemisphere, with a conducting flat base. The charge $q$ is still at the point $\mathbf{a}$. 

(e) Find the potential in the hemisphere.

(f) Find the vector force on the charge $q$. 

4. A plane wave of frequency $\omega$, with $\vec{E}_I = E_I \hat{j}$, is normally incident on a conducting plane of conductivity $\sigma$. The conducting plane fills the half space $z > 0$. The conductivity is very high, $\sigma \gg \varepsilon \omega$, so the displacement current inside the conductor can be neglected. There is a reflected wave, $\vec{E}_R$.

(a) Find the B and E fields inside the conductor in terms of their values at $z = 0$, as functions of $z$, $\sigma$ and $\omega$.

(b) Find the reflected electric field vector in terms of $E_I$, $\sigma$ and $\omega$.
5. An wire stretches along the z-axis from \( z = -\frac{a}{2} \) to \( z = \frac{a}{2} \). An alternating current of angular frequency \( \omega \) runs in the wire, and the radiated EM wave has a wavelength \( \lambda \) that is much greater than the wire length \( a \). A good approximation to the current density in the wire is

\[
\mathbf{J} = I_0 \delta(x) \delta(y) \mathbf{k} \left[ \frac{a}{2} - |z| \right] \sin(\omega t) \mathbf{k},
\]

where \( k = \frac{\omega}{c} \).

At large distances \( r \) from the wire, where \( r \gg a \), and \( r \gg \lambda \), find as functions of \( r, \theta, \phi, \omega \) and \( I_0 \):

(a) The vector potential \( A \).
(b) The magnetic field \( B \).
(c) The electric field \( E \).
(d) Find the power radiated per unit solid angle as a function of \( \theta, \phi, \omega \) and \( I_0 \).
(e) Find the wire's electric dipole moment \( \mathbf{p} \), and its magnetic dipole moment \( \mathbf{m} \).