University of Illinois at Chicago
Department of Physics

Classical Mechanics
Qualifying Examination

January 3, 2006
9:00 am-12:00 pm

Full credit can be achieved from completely correct answers to 4 questions. If the student attempts all 5 questions, all of the answers will be graded, and the top 4 scores will be counted toward the exam’s total score.
1. A toy consists of two concentric cylinders with inner radius $r$ and outer radius $R$. A string is wound around the inner radius and the outer radius can roll without slipping on a rough floor. The string is pulled at angle $\theta$ with respect to the horizontal.

   ![Free body diagram]

   a. Draw the free body diagram.
   b. Calculate the angular acceleration.
   c. Prove that there exists a critical angle $\theta_c$, where if $\theta < \theta_c$ the cylinder rolls away from the direction it is pulled, and if $\theta > \theta_c$ the cylinder rolls toward the direction it is pulled.
   d. Determine $\theta_c$

2. A positron $e^+$ with energy of 250 GeV/c$^2$ travels along the x axis and collides with a stationary electron. A single particle $V$ is produced and only $V$ remains after the collision. Later, $V$ decays into two identical mass ($m = 0.1$ GeV/c$^2$), unstable muons $\mu^+$ and $\mu^-$ which have lifetimes of $2 \times 10^{-6}$ s in their rest frame.

   a. Calculate the v/c of the positron.
   b. What is the mass of particle $V$?
   c. What is the total energy of the particle $V$ in its rest frame?
   d. What are the momenta of the electron and positron in the V rest frame?
   e. If the muon decays perpendicularly to the x axis in the V rest frame, what approximate angle does it make with respect to the x axis in the lab frame?
   f. How far would the muon travel in one lifetime as measured in the lab frame?
3. A particle of mass m moves in a field \( F = f(r)r \), where \( f(r) = -\frac{C}{r^3} \) and C > 0.
   
   a. Calculate \( \frac{dl}{dt} \), where \( l = mr^2 \frac{d\theta}{dt} \).
   
   b. Derive the equation of motion for \( r \) and show you can write it in form
      \[
      \frac{d^2u}{dt^2} + u = -\frac{m}{l^2u^2} f\left(\frac{1}{u}\right), \text{ where } u = \frac{1}{r}.\]
      Hint. Find the relationship of \( \frac{d}{d\theta} \) to \( \frac{d}{dt} \) for the central force.
   
   c. Show that a possible solution is spiral orbit of the form \( r = r_0 e^{\theta} \). Find all possible solutions.
   
   d. Show that \( \theta \) varies logarithmically with \( t \) for the spiral orbit from part c.
      Hint: integrate \( l \) to find \( \theta(t) \).

4. Two pendulums are coupled by a massless spring with spring constant \( k \). Both
   pendulums have massless springs of length \( L \). They are separated by distance \( D \). The
   masses are \( m \) and \( 2m \). Consider small oscillations.

   a. Solve for the normal modes of the pendulums.
   
   b. Determine the normal coordinates that undergo simple harmonic motion.
5. A bead of mass $m$ moves along a frictionless wire AB. The wire is fixed at point A and rotates with angular frequency $\omega$ about the z axis. $\theta$ is fixed

a. Determine the Lagrangian in terms of $r$, $\theta$ and azimuthal angle
b. Determine the Lagrange equation as a function of $m$, $\frac{dr}{dt}$, $\omega$, $r$ and $\theta$.
c. Solve the equation of motion.