Self-Study Problems

MCQ1: What is the hydrostatic (or gauge) pressure on an object submerged in water at a depth of 15 m at or near sea level?  
(A) $1.01 \times 10^5$ Pa  
(B) $1.47 \times 10^5$ Pa  
(C) $1.53 \times 10^5$ Pa  
(D) $2.48 \times 10^5$ Pa  

\[ P_{\text{gauge}} = \rho gh = 1.47 \times 10^5 \text{ Pa} \]

MCQ2: Figure to the right shows four situations in which a red liquid and gray liquid are in a U-tube. In one situation the liquids cannot be in static equilibrium. Which situation is that?  
(A) 1  
(B) 2  
(C) 3  
(D) 4  

\[ \rho_2 \Delta z = \rho_1 \Delta z \]

MCQ3: A 500 N weight sits on the small piston of a hydraulic machine. The small piston has area 2 cm\(^2\). If the large piston has area 40 cm\(^2\), how much weight can the large piston support?  
(A) 25 N  
(B) 500 N  
(C) 10000 N  
(D) 40000 N  

\[ F_1 = A_1 \frac{P_1}{\rho} \]

MCQ4: Figure to the right shows how stream of water emerging from a faucet "necks down" as it falls. The indicated cross-sectional areas are \( A_0 = 1.20 \text{ cm}^2 \) and \( A = 0.350 \text{ cm}^2 \). The two levels are separated by a vertical distance \( h = 45.0 \text{ mm} \). What is the volume flow rate (volume per unit of time) from the tap?  
(A) 28.1 cm\(^3\)/s  
(B) 34.4 cm\(^3\)/s  
(C) 43.2 cm\(^3\)/s  
(D) 57.7 cm\(^3\)/s  

\[ Q = A_0 v_0 = \frac{\pi R^4}{8 \eta} (\Delta p / L) \]

MCQ5: When inspecting the heating system of an old building, an engineer notices that many small pipes flow in parallel towards the furnace. She proposes that the small pipes be replaced by a larger pipe of twice the diameter. How many small pipes can the large one replace, and still maintain the same flow of fluid? Assume that the fluid is viscous, and that the pressure drop per unit length of pipe is the same in both cases.  
(A) 16  
(B) 8  
(C) 4  
(D) 32  
(E) 2  

\[ \frac{Q}{A} = \frac{\pi R^4}{8 \eta} (\Delta p / L) \]

Answer (A): According to Poiseuille's Law, the volume rate of flow \( Q \) in a cylinder of radius \( R \) is given by \( Q = \pi R^4 (\Delta p / 8 \eta L) \) where \( \eta \) is the fluid viscosity and \( \Delta p / L \) is the pressure gradient. Thus, if the radius is doubled, the flow increases by a factor of 2\(^4\), and 16 pipes can be replaced.
SP: The fresh water behind a reservoir dam has depth \( D = 15 \text{ m} \). A horizontal pipe of 4.0 cm in diameter passes through the dam at depth \( d = 6.0 \text{ m} \), and a plug secures the pipe opening as shown in the figure to the right.

(a) Find the magnitude of the frictional force between plug and pipe wall.

**Solution:**

Equilibrium condition: \( \sum F_i = 0 \). So, \( F_{\text{water}} - F_{\text{atm}} - f_s = 0 \)

where \( F_{\text{water}} = PA_{\text{pipe}}, \) \( P = P_{\text{atm}} + \rho gd, \) and

\[ F_{\text{atm}} = P_{\text{atm}}A_{\text{pipe}}, \] \( A_{\text{pipe}} = \pi r^2 \) (\( r = 2 \text{ cm} \))

\[ \Rightarrow (P_{\text{atm}} + \rho gd)A_{\text{pipe}} - P_{\text{atm}}A_{\text{pipe}} - f_s = 0 \Rightarrow f_s = \rho gdA_{\text{pipe}} = \rho gd\pi r^2 = 74 \text{ N} \]

(b) If the plug is removed, what water volume will exist the pipe in 3.0 h?

**Solution:** To find the volume owing out in one hour if the plug is removed, we will assume that the total volume of water in the reservoir is large enough that the change in the water level will be negligible. We then simply need to find the velocity of the water as it comes out the formerly plugged hole. So, we can apply Bernoulli’s equation, but we will keep in mind that \( v_{\text{top}} = 0 \) (the change in the water level will be negligible), and both \( P_{\text{top}} \) and \( P \) at the hole will be equal to \( P_{\text{atm}} \):

\[ P_{\text{top}} + \frac{1}{2} \rho v_{\text{top}}^2 + \rho gh_D = P + \frac{1}{2} \rho v_{\text{hole}}^2 + \rho g(D - d), \] \( \) where \( P_{\text{top}} = P = P_{\text{atm}} \) and \( v_{\text{top}} = 0 \)

\[ \Rightarrow v = v_{\text{hole}} = \sqrt{2gd} = 10.8 \text{ m/s} \]

To answer the question, first, we must calculate the volume flow rate as the product of the flow velocity and pipe area: \( Q = A_{\text{pipe}}v = \pi r^2 = 0.0136 \text{ m}^3/\text{s} \)

\[ \Rightarrow \text{The amount owing out in three hours} = Q/t = (0.0136 \text{ m}^3/\text{s})/(10800 \text{ s}) = 150 \text{ m}^3 \]

LP: Figure to the right shows a siphon, a device for removing liquid from a container. Tube must initially be filled, but once this has been done, liquid will flow through the tube until the liquid surface in the container is level with the tube opening at point A. If the container is filled with water, \( h_1 = 25 \text{ cm}, d = 12 \text{ cm} \) and \( h_2 = 40 \text{ cm} \),

(a) with what speed does the liquid emerge from the tube at C?

**Solution:** We consider a point D on the surface of the liquid in the container, in the same tube of flow with points A, B and C. Applying Bernoulli’s equation to points D and C, we obtain

\[ P_D + \frac{1}{2} \rho v_D^2 + \rho gh_D = P_C + \frac{1}{2} \rho v_C^2 + \rho gh_C \]

which leads to \( v_C = \sqrt{\frac{2(P_D-P_C)}{\rho} + 2g(h_D - h_C) + v_D^2} \)

Here, \( P_D = P_C = P_{\text{atm}}, h_D = d, h_C = -h_2 \) (A is the reference point) and \( v_D \approx 0 \)

\[ \Rightarrow v_C = \sqrt{2g(d + h_2)} = 3.2 \text{ m/s} \]

(b) If the atmospheric pressure is \( 1.01 \times 10^5 \text{ Pa} \), what is the pressure in the water at the topmost point B?

**Solution:** Bernoulli’s equation to points B and C:

\[ P_B + \frac{1}{2} \rho v_B^2 + \rho gh_B = P_C + \frac{1}{2} \rho v_C^2 + \rho gh_C \]

Here, \( v_B = v_C \) by equation of continuity, \( h_B = h_1 + d, h_C = -h_2 \) and \( P_C = P_{\text{atm}} \)

\[ P_B = P_{\text{atm}} + \rho g(h_C - h_B) = P_{\text{atm}} - \rho g(h_1 + h_2 + d) = 9.2 \times 10^4 \text{ Pa} \]

(c) What is the greatest possible height \( h_1 \) to which the siphon can lift water?

**Solution:** Since \( P_B \geq 0 \), then we must let \( P_{\text{atm}} - \rho g(h_1 + h_2 + d) \geq 0 \) which yields:

\[ h_1 \leq h_{1,\text{max}} = \frac{P_{\text{atm}}}{\rho g} - d - h_2 \leq \frac{P_{\text{atm}}}{\rho g} = 10.3 \text{ m} \]