SOLUTION

1 A Cube of Resistors

Suppose we have a “cube” of resistors (12 resistors total) all of the same resistance $R$ connected as in the figure to the left. Find the equivalent resistance between the two diagonally opposite corners (points a and b in the figure) in terms of $R$. (8 pts) Hint: Use symmetry.

In order to compute the resistance, we assume a voltage $V_{ab}$ is applied between points a and b and consider the currents that flow along each path between a and b. Let current $I$ enter at a and exit at b. At a there are three equivalent branches, so by symmetry the current must be $I/3$ at each.

At the next junction point there are two equivalent branches so, again by symmetry, each gets the current entering divided by two, i.e. $I/6$. Then at b there are three equivalent branches, which, again by symmetry, must each have current $I/3$. The voltage drop from a to b then is then given by taking the voltage drop over each resistor,

$$V_{ab} = \frac{I}{3}R + \frac{I}{6}R + \frac{I}{3}R = \frac{5}{6}IR$$

This must be the same as $V_{ab} = IR_{eq}$ for the equivalent resistor, so $R_{eq} = \frac{5}{6}R$. 
The neutron is a particle with zero net charge. However, it has a nonzero magnetic moment with magnitude equal to $9.66 \times 10^{-27}$ A · m$^2$. This can be explained by the internal structure of the neutron. The neutron is composed of three fundamental particles called quarks: an “up” ($u$) quark of charge $+2e/3$, and two “down” ($d$) quarks, each with charge $-e/3$. If the quarks are in motion, they can produce nonzero magnetic moment. As a very simple classical model, suppose the $u$ quark moves in a counter-clockwise circular path, and the two $d$ quarks move in a clockwise circular path, all with the same radius $r$ and speed $v$.

Note: To really model the neutron requires a strongly interacting quantum field theory known as Quantum Chromo-Dynamics, or QCD. Needless to say, this topic is slightly too advanced for us to get into here.

a) Determine the current due to the circulation of the $u$ quark, as well as the current due to each of the $d$ quarks (in terms of $e$, $r$ and $v$). (4 pts)

To obtain the current, we calculate the amount of charge passing a given point per unit time for the $u$ quark — since the quark is moving at constant speed $v$, $v = 2\pi r/T$, and so we have

$$I_u = \frac{q}{T} = \frac{2e}{3} \frac{v}{2\pi r} = \frac{ve}{3\pi r}$$

in the counter-clockwise direction. The computation is similar for the $d$ quarks, they are moving in the opposite direction with opposite sign of charge, so that for each down quark $I_d = ve/6\pi r$.

b) Determine the magnitude of the magnetic moment of the three quark system (in terms of $e$, $r$ and $v$). (4 pts)

The magnitude of the magnetic moment is given by $\mu = IA$, where we use the total current $I = I_u + 2I_d = 2ve/3\pi r$. The area is just $\pi r^2$, which gives

$$\mu = IA = \frac{2ve}{3\pi r} \pi r^2 = \frac{2ve}{3} r$$

c) Assuming that $r = 1.20 \times 10^{-15}$, determine the speed the quarks must be moving at to produce the magnetic moment of the neutron. (4 pts)

We have $\mu$, so we simply solve for $v$,

$$v = \frac{3\mu}{2er} = 7.55 \times 10^7 \text{ m/s}.$$