**MOTION ALONG A STRAIGHT LINE**

2.31. (a) **IDENTIFY** and **SET UP**: The acceleration \( a_x \) at time \( t \) is the slope of the tangent to the \( v_x \) versus \( t \) curve at time \( t \).

**EXECUTE**: At \( t = 3 \) s, the \( v_x \) versus \( t \) curve is a horizontal straight line, with zero slope. Thus \( a_x = 0 \).

At \( t = 7 \) s, the \( v_x \) versus \( t \) curve is a straight-line segment with slope \( \frac{45 \text{ m/s} - 20 \text{ m/s}}{9 \text{ s} - 5 \text{ s}} = 6.3 \text{ m/s}^2 \).

Thus \( a_x = 6.3 \text{ m/s}^2 \).

At \( t = 11 \) s the curve is again a straight-line segment, now with slope \( \frac{-0 - 45 \text{ m/s}}{13 \text{ s} - 9 \text{ s}} = -11.2 \text{ m/s}^2 \).

Thus \( a_x = -11.2 \text{ m/s}^2 \).

**EVALUATE**: \( a_x = 0 \) when \( v_x \) is constant, \( a_x > 0 \) when \( v_x \) is positive and the speed is increasing, and \( a_x < 0 \) when \( v_x \) is positive and the speed is decreasing.

(b) **IDENTIFY**: Calculate the displacement during the specified time interval.

**SET UP**: We can use the constant acceleration equations only for time intervals during which the acceleration is constant. If necessary, break the motion up into constant acceleration segments and apply the constant acceleration equations for each segment. For the time interval \( t = 0 \) to \( t = 5 \) s the acceleration is constant and equal to zero. For the time interval \( t = 5 \) s to \( t = 9 \) s the acceleration is constant and equal to \( 6.25 \text{ m/s}^2 \). For the interval \( t = 9 \) s to \( t = 13 \) s the acceleration is constant and equal to \( -11.2 \text{ m/s}^2 \).

**EXECUTE**: During the first 5 seconds the acceleration is constant, so the constant acceleration kinematic formulas can be used.

\[
v_{0x} = 20 \text{ m/s} \quad a_x = 0 \quad t = 5 \text{ s} \quad x - x_0 = ?
\]

\[
x - x_0 = v_{0x}t = 20 \text{ m/s}(5 \text{ s}) = 100 \text{ m}; \text{ this is the distance the officer travels in the first 5 seconds.}
\]

During the interval \( t = 5 \) s to \( t = 9 \) s the acceleration is again constant. The constant acceleration formulas can be applied to this 4-second interval. It is convenient to restart our clock so the interval starts at time \( t = 0 \) and ends at time \( t = 4 \) s. (Note that the acceleration is not constant over the entire \( t = 0 \) to \( t = 9 \) s interval.)

\[
v_{0x} = 20 \text{ m/s} \quad a_x = 6.25 \text{ m/s}^2 \quad t = 4 \text{ s} \quad x_0 = 100 \text{ m} \quad x - x_0 = ?
\]

\[
x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2
\]

\[
x - x_0 = (20 \text{ m/s})(4 \text{ s}) + \frac{1}{2}(6.25 \text{ m/s}^2)(4 \text{ s})^2 = 80 \text{ m} + 50 \text{ m} = 130 \text{ m}.
\]

Thus \( x - x_0 = 130 \text{ m} = 100 \text{ m} + 30 \text{ m} = 230 \text{ m} \).

At \( t = 9 \) s the officer is at \( x = 230 \text{ m} \), so she has traveled 230 m in the first 9 seconds.

During the interval \( t = 9 \) s to \( t = 13 \) s the acceleration is again constant. The constant acceleration formulas can be applied for this 4-second interval but not for the whole \( t = 0 \) to \( t = 13 \) s interval. To use the equations restart our clock so this interval begins at time \( t = 0 \) and ends at time \( t = 4 \) s.

\[
v_{0x} = 45 \text{ m/s} \text{ (at the start of this time interval)}
\]
\[ a_x = 2 \times 11.2 \text{ m/s}^2 \quad t = 4 \text{ s} \quad x_0 = 230 \text{ m} \quad x - x_0 = ? \]

\[ x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2 \]

\[ x - x_0 = (45 \text{ m/s})(4 \text{ s}) + \frac{1}{2}(-11.2 \text{ m/s}^2)(4 \text{ s})^2 = 180 \text{ m} - 89.6 \text{ m} = 90.4 \text{ m}. \]

Thus \( x = x_0 + 90.4 \text{ m} = 230 \text{ m} + 90.4 \text{ m} = 320 \text{ m}. \)

At \( t = 13 \text{ s} \) the officer is at \( x = 320 \text{ m} \), so she has traveled 320 m in the first 13 seconds.

**EVALUATE:** The velocity \( v_x \) is always positive so the displacement is always positive and displacement and distance traveled are the same. The average velocity for time interval \( \Delta t \) is \( \bar{v}_{av-x} = \Delta x / \Delta t \). For \( t = 0 \) to 5 s, \( \bar{v}_{av-x} = 20 \text{ m/s} \). For \( t = 0 \) to 9 s, \( \bar{v}_{av-x} = 26 \text{ m/s} \). For \( t = 0 \) to 13 s, \( \bar{v}_{av-x} = 25 \text{ m/s} \). These results are consistent with Figure 2.37 in the textbook.

### Section 2.34

**IDENTIFY:** Apply the constant acceleration equations to the motion of each vehicle. The truck passes the car when they are at the same \( x \) at the same \( t > 0 \).

**SET UP:** The truck has \( a_x = 0 \). The car has \( v_{0x} = 0 \). Let \( +x \) be in the direction of motion of the vehicles. Both vehicles start at \( x_0 = 0 \). The truck has \( a_x = 3.20 \text{ m/s}^2 \). The truck has \( v_x = 20.0 \text{ m/s} \).

**EXECUTE:** (a) \( x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2 \) gives \( x_T = v_{0T} t \) and \( x_C = \frac{1}{2} a_C t^2 \). Setting \( x_T = x_C \) gives \( t = 0 \) and \( v_{0T} = \frac{1}{2} a_C t \), so \( t = \frac{2v_{0T}}{a_C} = \frac{2(20.0 \text{ m/s})}{3.20 \text{ m/s}^2} = 12.5 \text{ s} \). At this time, \( x_T = (20.0 \text{ m/s})(12.5 \text{ s}) = 250 \text{ m} \) and \( x = \frac{1}{2}(3.20 \text{ m/s}^2)(12.5 \text{ s})^2 = 250 \text{ m} \). The car and truck have each traveled 250 m.

(b) At \( t = 12.5 \text{ s} \), the car has \( v_{x} = v_{0x} + a_x t = (3.20 \text{ m/s}^2)(12.5 \text{ s}) = 40 \text{ m/s} \).

(c) \( x_T = v_{0T} t \) and \( x_C = \frac{1}{2} a_C t^2 \). The \( x-t \) graph of the motion for each vehicle is sketched in Figure 2.34a.

(d) \( v_T = v_{0T} \). \( v_C = a_C t \). The \( v-x \) graph for each vehicle is sketched in Figure 2.34b.

**EVALUATE:** When the car overtakes the truck its speed is twice that of the truck.

![Figure 2.34a-b](image-url)

### Section 2.36

**IDENTIFY:** The rock has a constant downward acceleration of 9.80 m/s\(^2\). We know its initial velocity and position and its final position.

**SET UP:** We can use the kinematics formulas for constant acceleration.

**EXECUTE:** (a) \( y - y_0 = -30 \text{ m} \). \( v_{0y} = 18.0 \text{ m/s}, \quad a_y = -9.8 \text{ m/s}^2 \). The kinematics formulas give
\[
v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)} = \sqrt{(18.0 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)(-30 \text{ m})} = -30.2 \text{ m/s}, \quad \text{so the speed is 30.2 m/s.}
\]
2.47. **IDENTIFY:** We can avoid solving for the common height by considering the relation between height, time of fall and acceleration due to gravity and setting up a ratio involving time of fall and acceleration due to gravity.

**SET UP:** Let \( g \text{En} \) be the acceleration due to gravity on Enceladus and let \( g \) be this quantity on earth. Let \( h \) be the common height from which the object is dropped. Let \( \vec{y} \) be downward, so \( y - y_0 = h \). \( v_{0y} = 0 \)

**EXECUTE:** \( y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2 \) gives \( h = \frac{1}{2} g \text{En}^2 \) and \( h = \frac{1}{2} g \text{En}^2 \text{En} \). Combining these two equations gives \( g \text{En}^2 = g \text{En} \text{En}^2 \) and \( g \text{En} = \left( \frac{a_E}{\text{En}} \right)^2 = (9.80 \text{ m/s}^2 \left( \frac{1.75 \text{ s}}{18.6 \text{ s}} \right)^2 = 0.0868 \text{ m/s}^2 \).

**EVALUATE:** The acceleration due to gravity is inversely proportional to the square of the time of fall.

2.76. **IDENTIFY:** The acceleration is not constant so the constant acceleration equations cannot be used. Instead, use \( a_x(t) = \frac{dv_x}{dt} \) and \( x = x_0 + \int a_x(t)dt \).

**SET UP:** \( \int a dt = \frac{1}{n+1} t^{n+1} \) for \( n \geq 0 \).

**EXECUTE:** (a) \( x(t) = x_0 + \int \left( a - \frac{\beta t^2}{2} \right) dt = x_0 + \alpha t - \frac{\beta}{2} t^3 \). \( x = 0 \) at \( t = 0 \) gives \( x_0 = 0 \) and \( x(t) = \alpha t - \frac{\beta}{2} t^3 = (4,00 \text{ m/s})t - (0.667 \text{ m/s}^3)t^3 \). \( a_x(t) = \frac{dv_x}{dt} = -2 \beta t = -(4,00 \text{ m/s}^3)t \).

(b) The maximum positive \( x \) is when \( v_x = 0 \) and \( a_x < 0 \). \( v_x = 0 \) gives \( \alpha - \beta t^2 = 0 \) and \( t = \sqrt[3]{\frac{2 \alpha}{\beta}} = 1.41 \text{ s} \). At this \( t \), \( a_x \) is negative. For \( t = 1.41 \text{ s} \), \( x = (4.00 \text{ m/s})(1.41 \text{ s}) - (0.667 \text{ m/s}^3)(1.41 \text{ s})^3 = 3.77 \text{ m} \).

**EVALUATE:** After \( t = 1.41 \text{ s} \) the object starts to move in the \(-x \) direction and goes to \( x = -\infty \) as \( t \to \infty \).

2.87. **IDENTIFY and SET UP:** Let \( +y \) be upward. Each ball moves with constant acceleration \( a_y = -9.80 \text{ m/s}^2 \). In parts (c) and (d) require that the two balls be at the same height at the same time.

**EXECUTE:** (a) At ceiling, \( v_y = 0 \), \( y - y_0 = 3.0 \text{ m} \), \( a_y = -9.80 \text{ m/s}^2 \). Solve for \( v_{0y} \).

\[ v_y = v_{0y}^2 + 2a_y(y - y_0) \text{ gives } v_{0y} = 7.7 \text{ m/s} \]

(b) \( v_y = v_{0y} + a_y t \) with the information from part (a) gives \( t = 0.78 \text{ s} \).

(c) Let the first ball travel downward a distance \( d \) in time \( t \). It starts from its maximum height, so \( v_{0y} = 0 \).

\[ y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2 \text{ gives } d = (4.9 \text{ m/s}^2) t^2 \]

The second ball has \( v_{0y} = \frac{2}{3}(7.7 \text{ m/s}) = 5.1 \text{ m/s} \). In time \( t \) it must travel upward \( 3.0 \text{ m} - d \) to be at the same place as the first ball.

\[ y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2 \text{ gives } 3.0 \text{ m} - d = (5.1 \text{ m/s})t - (4.9 \text{ m/s}^2)t^2 \]

We have two equations in two unknowns, \( d \) and \( t \). Solving gives \( t = 0.59 \text{ s} \) and \( d = 1.7 \text{ m} \).

(d) \( 3.0 \text{ m} - d = 1.3 \text{ m} \)

**EVALUATE:** In 0.59 s the first ball falls \( d = (4.9 \text{ m/s}^2)(0.59 \text{ s})^2 = 1.7 \text{ m} \), so is at the same height as the second ball.
Motion in Two or Three Dimensions

2.93. **IDENTIFY:** Apply constant acceleration equations to the motion of the rocket and to the motion of the canister after it is released. Find the time it takes the canister to reach the ground after it is released and find the height of the rocket after this time has elapsed. The canister travels up to its maximum height and then returns to the ground.

**SET UP:** Let $+y$ be upward. At the instant that the canister is released, it has the same velocity as the rocket. After it is released, the canister has $a_y = -9.80 \text{ m/s}^2$. At its maximum height the canister has $v_y = 0$.

**EXECUTE:** (a) Find the speed of the rocket when the canister is released: $v_{0y} = 0$, $a_x = 3.30 \text{ m/s}^2$, $y - y_0 = 235 \text{ m}$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(3.30 \text{ m/s}^2)(235 \text{ m})} = 39.4 \text{ m/s}$.

For the motion of the canister after it is released, $v_{0y} = +39.4 \text{ m/s}$, $a_x = -9.80 \text{ m/s}^2$. $y - y_0 = -235 \text{ m}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_xt^2$ gives $-235 \text{ m} = (39.4 \text{ m/s})t - \left(\frac{9.80 \text{ m/s}^2}{2}\right)t^2$. The quadratic formula gives $t = 12.0 \text{ s}$ as the positive solution. Then for the motion of the rocket during this $12.0 \text{ s}, y - y_0 = v_{0y}t + \frac{1}{2}a_xt^2 = 235 \text{ m} + (39.4 \text{ m/s})(12.0 \text{ s}) + \frac{1}{2}(3.30 \text{ m/s}^2)(12.0 \text{ s})^2 = 945 \text{ m}$.

(b) Find the maximum height of the canister above its release point: $v_{0y} = +39.4 \text{ m/s}$, $v_y = 0$,

$a_y = -9.80 \text{ m/s}^2$, $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (39.4 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 79.2 \text{ m}$. After its release the canister travels upward 79.2 m to its maximum height and then back down 79.2 m + 235 m to the ground. The total distance it travels is 393 m.

**EVALUATE:** The speed of the rocket at the instant that the canister returns to the launch pad is $v_y = v_{0y} + a_yt = 39.4 \text{ m/s} + (3.30 \text{ m/s}^2)(12.0 \text{ s}) = 79.0 \text{ m/s}$. We can calculate its height at this instant by $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ with $v_{0y} = 0$ and $v_y = 79.0 \text{ m/s}$. $y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{(79.0 \text{ m/s})^2}{2(3.30 \text{ m/s}^2)} = 946 \text{ m}$, which agrees with our previous calculation.

3.10. **IDENTIFY:** The person moves in projectile motion. She must travel 1.75 m horizontally during the time she falls 9.00 m vertically.

**SET UP:** Take $+y$ downward. $a_x = 0$, $a_y = -9.80 \text{ m/s}^2$. $v_{0x} = v_0$, $v_{0y} = 0$. 


**EXECUTE:** Time to fall 9.00 m:  
\[ y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 \]  
gives  
\[ t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(9.00 \text{ m})}{9.80 \text{ m/s}^2}} = 1.36 \text{ s}. \]

Speed needed to travel 1.75 m horizontally during this time:  
\[ x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \]  
gives  
\[ v_{0x} = \frac{x - x_0}{t} = \frac{1.75 \text{ m}}{1.36 \text{ s}} = 1.29 \text{ m/s}. \]

**EVALUATE:** If she increases her initial speed she still takes 1.36 s to reach the level of the ledge, but has traveled horizontally farther than 1.75 m.

### 3.21. IDENTIFY:

Take the origin of coordinates at the roof and let the \( +y \)-direction be upward. The rock moves in projectile motion, with \( a_x = 0 \) and \( a_y = -g \). Apply constant acceleration equations for the \( x \) and \( y \) components of the motion.

**SET UP:**

![Figure 3.21a](image)

(a) At the maximum height \( v_y = 0 \).

\[ a_y = -9.80 \text{ m/s}^2, \quad v_y = 0, \quad v_{0y} = +16.3 \text{ m/s}, \quad y - y_0 = ? \]

\[ v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \]

**EXECUTE:**  
\[ y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (16.3 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = +13.6 \text{ m} \]

(b) **SET UP:** Find the velocity by solving for its \( x \) and \( y \) components.

\[ v_x = v_{0x} = 25.2 \text{ m/s} \quad (\text{since} \quad a_x = 0) \]

\[ v_y = ?, \quad a_y = -9.80 \text{ m/s}^2, \quad y - y_0 = -15.0 \text{ m} \quad (\text{negative because at the ground the rock is below its initial position}), \quad v_{0y} = 16.3 \text{ m/s} \]

\[ v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \]

\[ v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)} \quad (v_y \text{ is negative because at the ground the rock is traveling downward.}) \]

**EXECUTE:**  
\[ v_y = -\sqrt{(16.3 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-15.0 \text{ m})} = -23.7 \text{ m/s} \]

Then  
\[ v = \sqrt{v_x^2 + v_y^2} = \sqrt{(25.2 \text{ m/s})^2 + (-23.7 \text{ m/s})^2} = 34.6 \text{ m/s}, \]

(c) **SET UP:** Use the vertical motion (\( y \)-component) to find the time the rock is in the air:

\[ t = ?, \quad v_y = -23.7 \text{ m/s} \quad (\text{from part (b)}), \quad a_y = -9.80 \text{ m/s}^2, \quad v_{0y} = +16.3 \text{ m/s} \]

**EXECUTE:**  
\[ t = \frac{v_y - v_{0y}}{a_y} = \frac{-23.7 \text{ m/s} - 16.3 \text{ m/s}}{-9.80 \text{ m/s}^2} = +4.08 \text{ s} \]
SET UP: Can use this $t$ to calculate the horizontal range:

\[ t = 4.08 \text{ s}, \quad v_{0x} = 25.2 \text{ m/s}, \quad a_x = 0, \quad x - x_0 = ? \]

EXECUTE: 

\[ x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (25.2 \text{ m/s})(4.08 \text{ s}) + 0 = 103 \text{ m} \]

(d) Graphs of $x$ versus $t$, $y$ versus $t$, $v_x$ versus $t$ and $v_y$ versus $t$:

![Graphs of x versus t, y versus t, v_x versus t, and v_y versus t](image)

**Figure 3.21b**

EVALUATE: The time it takes the rock to travel vertically to the ground is the time it has to travel horizontally. With $v_{0y} = +16.3 \text{ m/s}$ the time it takes the rock to return to the level of the roof ($y = 0$) is

\[ t = 2v_{0y}/g = 3.33 \text{ s}. \]

The time in the air is greater than this because the rock travels an additional 15.0 m to the ground.

EVALUATE: The acceleration is large and the force on the fluid must be 2.5 times its weight.

3.25. IDENTIFY: Apply Eq. (3.30).

SET UP: \[ T = 24 \text{ h}. \]

EXECUTE: 

(a) \[ a_{rad} = \frac{4\pi^2 (6.38 \times 10^6 \text{ m})}{((24 \text{ h})(3600 \text{ s/h}))^2} = 0.034 \text{ m/s}^2 = 3.4 \times 10^{-3} \text{ g}. \]

(b) Solving Eq. (3.30) for the period $T$ with $a_{rad} = g$, \[ T = \sqrt{\frac{4\pi^2 (6.38 \times 10^6 \text{ m})}{9.80 \text{ m/s}^2}} = 5070 \text{ s} = 1.4 \text{ h}. \]

EVALUATE: \[ a_{rad} \text{ is proportional to } 1/T^2, \text{ so to increase } a_{rad} \text{ by a factor of } \frac{1}{3.4 \times 10^{-3}} = 294 \text{ requires that } T \text{ be multiplied by a factor of } \frac{24 \text{ h}}{\sqrt{294}} = 1.4 \text{ h}. \]

3.53. IDENTIFY: The canister moves in projectile motion. Its initial velocity is horizontal. Apply constant acceleration equations for the $x$ and $y$ components of motion.

SET UP:

![Diagram](image)

Take the origin of coordinates at the point where the canister is released. Take +y to be upward. The initial velocity of the canister is the velocity of the plane, 64.0 m/s in the +x-direction.

**Figure 3.53**

Use the vertical motion to find the time of fall:

\[ t = ?, \quad v_{0y} = 0, \quad a_y = -9.80 \text{ m/s}^2, \quad y - y_0 = -90.0 \text{ m} \text{ (When the canister reaches the ground it is 90.0 m below the origin.)} \]

\[ y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 \]
EXECUTE: Since $v_{0y} = 0$, \[ t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(-90.0 \text{ m})}{-9.80 \text{ m/s}^2}} = 4.286 \text{ s}. \]

SET UP: Then use the horizontal component of the motion to calculate how far the canister falls in this time:

\[ x - x_0 = ?', \quad a_x = 0, \quad v_{0x} = 64.0 \text{ m/s} \]

EXECUTE: \[ x - x_0 = v_{0x} + \frac{1}{2}a_xt^2 = (64.0 \text{ m/s})(4.286 \text{ s}) + 0 = 274 \text{ m}. \]

EVALUATE: The time it takes the canister to fall 90.0 m, starting from rest, is the time it travels horizontally at constant speed.

3.81. IDENTIFY: Relative velocity problem. The plane’s motion relative to the earth is determined by its velocity relative to the earth.

SET UP: Select a coordinate system where $+y$ is north and $+x$ is east.

The velocity vectors in the problem are:

- $\vec{v}_{\text{P/E}}$, the velocity of the plane relative to the earth.
- $\vec{v}_{\text{P/A}}$, the velocity of the plane relative to the air (the magnitude $v_{\text{P/A}}$ is the airspeed of the plane and the direction of $\vec{v}_{\text{P/A}}$ is the compass course set by the pilot).
- $\vec{v}_{\text{A/E}}$, the velocity of the air relative to the earth (the wind velocity).

The rule for combining relative velocities gives $\vec{v}_{\text{P/E}} = \vec{v}_{\text{P/A}} + \vec{v}_{\text{A/E}}$.

(a) We are given the following information about the relative velocities:

$\vec{v}_{\text{P/A}}$ has magnitude 220 km/h and its direction is west. In our coordinates it has components

\[ (v_{\text{P/A}})_x = -220 \text{ km/h} \quad \text{and} \quad (v_{\text{P/A}})_y = 0. \]

From the displacement of the plane relative to the earth after 0.500 h, we find that $\vec{v}_{\text{P/E}}$ has components in our coordinate system of

\[ (v_{\text{P/E}})_x = \frac{120 \text{ km}}{0.500 \text{ h}} = -240 \text{ km/h (west)} \]

\[ (v_{\text{P/E}})_y = \frac{-20 \text{ km}}{0.500 \text{ h}} = -40 \text{ km/h (south)} \]

With this information the diagram corresponding to the velocity addition equation is shown in Figure 3.81a.

![Figure 3.81a](image1)

We are asked to find $\vec{v}_{\text{A/E}}$, so solve for this vector:

$\vec{v}_{\text{P/E}} = \vec{v}_{\text{P/A}} + \vec{v}_{\text{A/E}}$ gives $\vec{v}_{\text{A/E}} = \vec{v}_{\text{P/E}} - \vec{v}_{\text{P/A}}$.

EXECUTE: The $x$-component of this equation gives

\[ (v_{\text{A/E}})_x = (v_{\text{P/E}})_x - (v_{\text{P/A}})_x = -240 \text{ km/h} - (-220 \text{ km/h}) = -20 \text{ km/h}. \]

The $y$-component of this equation gives

\[ (v_{\text{A/E}})_y = (v_{\text{P/E}})_y - (v_{\text{P/A}})_y = -40 \text{ km/h}. \]

Now that we have the components of $\vec{v}_{\text{A/E}}$ we can find its magnitude and direction.
\[ v_{AE} = \sqrt{(v_{A/E})_x^2 + (v_{A/E})_y^2} \]
\[ v_{AE} = \sqrt{(-20 \text{ km/h})^2 + (-40 \text{ km/h})^2} = 44.7 \text{ km/h} \]
\[ \tan \phi = \frac{40 \text{ km/h}}{20 \text{ km/h}} = 2.00; \quad \phi = 63.4^\circ \]

The direction of the wind velocity is 63.4° S of W, or 26.6° W of S.

**Figure 3.81b**

**EVALUATE:** The plane heads west. It goes farther west than it would without wind and also travels south, so the wind velocity has components west and south.

**(b) SET UP:** The rule for combining the relative velocities is still \( \vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E} \), but some of these velocities have different values than in part (a).

\( \vec{v}_{P/A} \) has magnitude 220 km/h but its direction is to be found.

\( \vec{v}_{A/E} \) has magnitude 40 km/h and its direction is due south.

The direction of \( \vec{v}_{P/E} \) is west; its magnitude is not given.

The vector diagram for \( \vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E} \) and the specified directions for the vectors is shown in Figure 3.81c.

**Figure 3.81c**

The vector addition diagram forms a right triangle.

**EXECUTE:**
\[ \sin \phi = \frac{v_{AE}}{v_{P/A}} = \frac{40 \text{ km/h}}{220 \text{ km/h}} = 0.1818; \quad \phi = 10.5^\circ. \]

The pilot should set her course 10.5° north of west.

**EVALUATE:** The velocity of the plane relative to the air must have a northward component to counteract the wind and a westward component in order to travel west.