Goals for Chapter 5

• To use Newton’s first law for bodies in equilibrium

• To use Newton’s second law for accelerating bodies

• To study the types of friction and fluid resistance

• To solve problems involving circular motion
Introduction

- We’ll extend the problem-solving skills we began to develop in Chapter 4.

- We’ll start with equilibrium, in which a body is at rest or moving with constant velocity.

- Next, we’ll study objects that are not in equilibrium and deal with the relationship between forces and motion.

- We’ll analyze the friction force that acts when a body slides over a surface.

- We’ll analyze the forces on a body in circular motion at constant speed.
Using Newton’s First Law when forces are in equilibrium

- A body is in equilibrium when it is at rest or moving with constant velocity in an inertial frame of reference.

- Follow Problem-Solving Strategy 5.1.

**Problem-Solving Strategy 5.1**

<table>
<thead>
<tr>
<th>Newton’s First Law: Equilibrium of a Particle</th>
</tr>
</thead>
</table>
| **IDENTIFY** the relevant concepts:** You must use Newton’s first law for any problem that involves forces acting on a body in equilibrium—that is, either at rest or moving with constant velocity. For example, a car is in equilibrium when it’s parked, but also when it’s traveling down a straight road at a steady speed.

If the problem involves more than one body and the bodies interact with each other, you’ll also need to use Newton’s third law. This law allows you to relate the force that one body exerts on a second body to the force that the second body exerts on the first one.

Identify the target variable(s). Common target variables in equilibrium problems include the magnitude and direction (angle) of one of the forces, or the components of a force.

**SET UP** the problem using the following steps:

1. Draw a very simple sketch of the physical situation, showing dimensions and angles. You don’t have to be an artist!
2. Draw a free-body diagram for each body that is in equilibrium. For the present, we consider the body as a particle, so you can represent it as a large dot. In your free-body diagram, do **not** include the other bodies that interact with it, such as a surface it may be resting on or a rope pulling on it.
3. Ask yourself what is interacting with the body by touching it or in any other way. On your free-body diagram, draw a force vector for each interaction. Label each force with a symbol for the magnitude of the force. If you know the angle at which a force is directed, draw the angle accurately and label it. Include the body’s weight, unless the body has negligible mass. If the mass is given, use \( w = mg \) to find the weight. A surface in contact with the body exerts a normal force perpendicular to the surface and possibly a friction force parallel to the surface. A rope or chain exerts a pull (never a push) in a direction along its length.
4. **Do not** show in the free-body diagram any forces exerted by the body on any other body. The sums in Eqs. (5.1) and (5.2) include only forces that act on the body. For each force on the body, ask yourself “What other body causes that force?” If you can’t answer that question, you may be imagining a force that isn’t there.
5. Choose a set of coordinate axes and include them in your free-body diagram. (If there is more than one body in the problem, choose axes for each body separately.) Label the positive direction for each axis. If a body rests or slides on a plane surface, it usually simplifies things to choose axes that are parallel and perpendicular to this surface, even when the plane is tilted.

**EXECUTE** the solution as follows:

1. Find the components of each force along each of the body’s coordinate axes. Draw a wiggly line through each force vector that has been replaced by its components, so you don’t count it twice. The **magnitude** of a force is always positive, but its **components** may be positive or negative.
2. Set the sum of all \( x \)-components of force equal to zero. In a separate equation, set the sum of all \( y \)-components equal to zero. (*Never* add \( x \)- and \( y \)-components in a single equation.)
3. If there are two or more bodies, repeat all of the above steps for each body. If the bodies interact with each other, use Newton’s third law to relate the forces they exert on each other.
4. Make sure that you have as many independent equations as the number of unknown quantities. Then solve these equations to obtain the target variables.

**EVALUATE** your answer: Look at your results and ask whether they make sense. When the result is a symbolic expression or formula, check to see that your formula works for any special cases (particular values or extreme cases for the various quantities) for which you can guess what the results ought to be.
One-dimensional equilibrium: Tension in a massless rope

- A gymnast hangs from the end of a massless rope.
- Follow Example 5.1.

(a) The situation

(b) Free-body diagram for gymnast

(c) Free-body diagram for rope

Action–reaction pair
One-dimensional equilibrium: Tension in a rope with mass

- What is the tension in the previous example if the rope has mass?
- Follow Example 5.2.
Two-dimensional equilibrium

- A car engine hangs from several chains.
- Follow Example 5.3.

(a) Engine, chains, and ring

(b) Free-body diagram for engine

(c) Free-body diagram for ring O
A car on an inclined plane

- An car rests on a slanted ramp.
- Follow Example 5.4.

(a) Car on ramp

(b) Free-body diagram for car

We replace the weight by its components.

\[
\begin{align*}
\text{Weight} & = w \\
\text{Normal force} & = n \\
\text{Tension} & = T \\
\text{Component along ramp} & = w \sin \alpha \\
\text{Component normal to ramp} & = w \cos \alpha
\end{align*}
\]
Bodies connected by a cable and pulley

- A cart is connected to a bucket by a cable passing over a pulley.
- Draw separate free-body diagrams for the bucket and the cart.
- Follow Example 5.5.
Using Newton’s Second Law: Dynamics of Particles

• Apply Newton’s second law in component form.

\[ \Sigma F_x = ma_x \quad \Sigma F_y = ma_y \]

• Follow Problem-Solving Strategy 5.2.

**Problem-Solving Strategy 5.2**

**IDENTIFY** the relevant concepts: You have to use Newton’s second law for any problem that involves forces acting on an accelerating body.

Identify the target variable—usually an acceleration or a force. If the target variable is something else, you’ll need to select another concept to use. For example, suppose the target variable is how fast a sled is moving when it reaches the bottom of a hill. Newton’s second law will let you find the sled’s acceleration; you’ll then use the constant-acceleration relationships from Section 2.4 to find velocity from acceleration.

**SET UP** the problem using the following steps:

1. Draw a simple sketch of the situation that shows each moving body. For each body, draw a free-body diagram that shows all the forces acting on the body. (The acceleration of a body is determined by the forces that act on it, not by the forces that it exerts on anything else.) Make sure you can answer the question “What other body is applying this force?” for each force in your diagram. Never include the quantity \( m\ddot{a} \) in your free-body diagram; it’s not a force!
2. Label each force with an algebraic symbol for the force’s magnitude. Usually, one of the forces will be the body’s weight; it’s usually best to label this as \( w = mg \).
3. Choose your \( x \)- and \( y \)-coordinate axes for each body, and show them in its free-body diagram. Be sure to indicate the positive direction for each axis. If you know the direction of the acceleration, it usually simplifies things to take one positive axis along that direction. If your problem involves two or more bodies that accelerate in different directions, you can use a different set of axes for each body.
4. In addition to Newton’s second law, \( \Sigma \vec{F} = m\ddot{a} \), identify any other equations you might need. For example, you might need one or more of the equations for motion with constant acceleration. If more than one body is involved, there may be relationships among their motions; for example, they may be connected by a rope. Express any such relationships as equations relating the accelerations of the various bodies.

**EXECUTE** the solution as follows:

1. For each body, determine the components of the forces along each of the body’s coordinate axes. When you represent a force in terms of its components, draw a wiggly line through the original force vector to remind you not to include it twice.
2. Make a list of all the known and unknown quantities. In your list, identify the target variable or variables.
3. For each body, write a separate equation for each component of Newton’s second law, as in Eqs. (5.4). In addition, write any additional equations that you identified in step 4 of “Set Up.” (You need as many equations as there are target variables.)
4. Do the easy part—the math! Solve the equations to find the target variable(s).

**EVALUATE** your answer: Does your answer have the correct units? (When appropriate, use the conversion 1 N = 1 kg \cdot m/s^2.) Does it have the correct algebraic sign? When possible, consider particular values or extreme cases of quantities and compare the results with your intuitive expectations. Ask, “Does this result make sense?”
A note on free-body diagrams

• Refer to Figure 5.6.

• Only the force of gravity acts on the falling apple.

• \( \vec{m}a \) does not belong in a free-body diagram.
Straight-line motion with constant force

- The wind exerts a constant horizontal force on the boat.
- Follow Example 5.6.

(a) Iceboat and rider on frictionless ice
(b) Free-body diagram for iceboat and rider

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Straight-line motion with friction

• For the ice boat in the previous example, a constant horizontal friction force now opposes its motion.

• Follow Example 5.7.
Tension in an elevator cable

• The elevator is moving downward but slowing to a stop.
• What is the tension in the supporting cable?
• Follow Example 5.8.
Apparent weight in an accelerating elevator

• A woman inside the elevator of the previous example is standing on a scale. How will the acceleration of the elevator affect the scale reading?

• Follow Example 5.9.

(a) Woman in a descending elevator

(b) Free-body diagram for woman

Moving down with decreasing speed

\[ w = 490 \, \text{N} \]
Acceleration down a hill

• What is the acceleration of a toboggan sliding down a friction-free slope? Follow Example 5.10.

(a) The situation

(b) Free-body diagram for toboggan
Two common free-body diagram errors

- The normal force must be perpendicular to the surface.
- There is no \( \vec{ma} \) force.
- See Figure 5.13.

(a) Correct free-body diagram for the sled

- Normal force is perpendicular to the surface.
- It’s OK to draw the acceleration vector adjacent to (but not touching) the body.
- \( \vec{W} = mg \)

(b) Incorrect free-body diagram for the sled

- Normal force is not vertical because the surface (which is along the \( x \)-axis) is inclined.
- The quantity \( \vec{ma} \) is not a force.
- \( \vec{W} = mg \)
Two bodies with the same acceleration

- We can treat the milk carton and tray as *separate bodies*, or we can treat them as a single *composite body*.

- Follow Example 5.11.

(a) A milk carton and a food tray

(b) Free-body diagram for milk carton

(c) Free-body diagram for food tray

(d) Free-body diagram for carton and tray as a composite body
Two bodies with the same magnitude of acceleration

- The glider on the air track and the falling weight move in different directions, but their accelerations have the same magnitude.
- Follow Example 5.12 using Figure 5.15.
Frictional forces

- When a body rests or slides on a surface, the *friction force* is parallel to the surface.

- Friction between two surfaces arises from interactions between molecules on the surfaces.
Kinetic and static friction

- **Kinetic friction** acts when a body slides over a surface.

- The *kinetic friction force* is $f_k = \mu_k n$.

- **Static friction** acts when there is no relative motion between bodies.

- The *static friction force* can vary between zero and its maximum value: $f_s \leq \mu_s n$. 
Static friction followed by kinetic friction

- Before the box slides, static friction acts. But once it starts to slide, kinetic friction acts.
Some approximate coefficients of friction

**Table 5.1 Approximate Coefficients of Friction**

<table>
<thead>
<tr>
<th>Materials</th>
<th>Coefficient of Static Friction, $\mu_s$</th>
<th>Coefficient of Kinetic Friction, $\mu_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel on steel</td>
<td>0.74</td>
<td>0.57</td>
</tr>
<tr>
<td>Aluminum on steel</td>
<td>0.61</td>
<td>0.47</td>
</tr>
<tr>
<td>Copper on steel</td>
<td>0.53</td>
<td>0.36</td>
</tr>
<tr>
<td>Brass on steel</td>
<td>0.51</td>
<td>0.44</td>
</tr>
<tr>
<td>Zinc on cast iron</td>
<td>0.85</td>
<td>0.21</td>
</tr>
<tr>
<td>Copper on cast iron</td>
<td>1.05</td>
<td>0.29</td>
</tr>
<tr>
<td>Glass on glass</td>
<td>0.94</td>
<td>0.40</td>
</tr>
<tr>
<td>Copper on glass</td>
<td>0.68</td>
<td>0.53</td>
</tr>
<tr>
<td>Teflon on Teflon</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Teflon on steel</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Rubber on concrete (dry)</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td>Rubber on concrete (wet)</td>
<td>0.30</td>
<td>0.25</td>
</tr>
</tbody>
</table>

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Friction in horizontal motion

- Before the crate moves, static friction acts on it. After it starts to move, kinetic friction acts.

- Follow Example 5.13.
Static friction can be less than the maximum

- Static friction only has its maximum value just before the box “breaks loose” and starts to slide.
- Follow Example 5.14.
Pulling a crate at an angle

- The angle of the pull affects the normal force, which in turn affects the friction force.

- Follow Example 5.15.
Motion on a slope having friction

- Consider the toboggan from Example 5.10, but with friction. Follow Example 5.16 and Figure 5.22.

- Consider the toboggan on a steeper hill, so it is now accelerating. Follow Example 5.17 and Figure 5.23.
Fluid resistance and terminal speed

- The fluid resistance on a body depends on the speed of the body.
- A falling body reaches its terminal speed when the resisting force equals the weight of the body.
- The figures at the right illustrate the effects of air drag.
- Follow Example 5.18.
Dynamics of circular motion

• If a particle is in uniform circular motion, both its acceleration and the net force on it are directed toward the center of the circle.

• The net force on the particle is $F_{\text{net}} = \frac{mv^2}{R}$. 

In uniform circular motion, the acceleration and net force are both directed toward the center of the circle.
What if the string breaks?

• If the string breaks, no net force acts on the ball, so it obeys Newton’s first law and moves in a straight line.
Avoid using “centrifugal force”

- Figure (a) shows the correct free-body diagram for a body in uniform circular motion.

- Figure (b) shows a common error.

- In an inertial frame of reference, there is no such thing as “centrifugal force.”
Force in uniform circular motion

• A sled on frictionless ice is kept in uniform circular motion by a rope.

• Follow Example 5.19.

(a) A sled in uniform circular motion

(b) Free-body diagram for sled

We point the positive $x$-direction toward the center of the circle.
A conical pendulum

- A bob at the end of a wire moves in a horizontal circle with constant speed.
- Follow Example 5.20.

(a) The situation

(b) Free-body diagram for pendulum bob

We point the positive x-direction toward the center of the circle.
A car rounds a flat curve

- A car rounds a flat unbanked curve. What is its maximum speed?
- Follow Example 5.21.

(a) Car rounding flat curve

(b) Free-body diagram for car

\[ w = mg \]

\[ q_{rad} \]
A car rounds a banked curve

• At what angle should a curve be banked so a car can make the turn even with no friction?

• Follow Example 5.22.

(a) Car rounding banked curve

(b) Free-body diagram for car

\[ \begin{align*}
\text{normal force} & = n \\
\text{normal force projection} & = n \cos \beta \\
\text{normal force projection} & = n \sin \beta \\
\text{acceleration} & = a_{rad}
\end{align*} \]

\[ w = mg \]
Uniform motion in a vertical circle

- A person on a Ferris wheel moves in a vertical circle.
- Follow Example 5.23.

(a) Sketch of two positions

(b) Free-body diagram for passenger at top

(c) Free-body diagram for passenger at bottom
The fundamental forces of nature

- According to current understanding, all forces are expressions of four distinct fundamental forces:
  - gravitational interactions
  - electromagnetic interactions
  - the strong interaction
  - the weak interaction
- Physicists have taken steps to unify all interactions into a theory of everything.