Goals for Chapter 11

• To study the conditions for equilibrium of a body

• To understand center of gravity and how it relates to a body’s stability

• To solve problems for rigid bodies in equilibrium

• To analyze situations involving tension, compression, pressure, and shear

• To investigate what happens when a body is stretched so much that it deforms or breaks
Introduction

• Many bodies, such as bridges, aqueducts, and ladders, are designed so they do not accelerate.

• Real materials are not truly rigid. They are elastic and do deform to some extent.

• We shall introduce concepts such as stress and strain to understand the deformation of real bodies.
Conditions for equilibrium

- **First condition**: The sum of all the forces is equal to zero:
  \[ \Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0 \]

- **Second condition**: The sum of all torques about any given point is equal to zero.

(a) This body is in static equilibrium.

(b) This body has no tendency to accelerate as a whole, but it has a tendency to start rotating.

First condition satisfied:
Net force = 0, so body at rest has no tendency to start moving as a whole.

Second condition NOT satisfied: There is a net clockwise torque about the axis, so body at rest will start rotating clockwise.

(c) This body has a tendency to accelerate as a whole but no tendency to start rotating.

First condition NOT satisfied: There is a net upward force, so body at rest will start moving upward.

Second condition satisfied: Net torque about the axis = 0, so body at rest has no tendency to start rotating.
Center of gravity

- We can treat a body’s weight as though it all acts at a single point—the center of gravity.

- If we can ignore the variation of gravity with altitude, the center of gravity is the same as the center of mass.
Walking the plank

- Follow Example 11.1 using Figure 11.6 below.

\[ L = 6.0\, \text{m} \]
\[ D = 1.5\, \text{m} \]
\[ \frac{L}{2} - \frac{D}{2} \]

\[ M = 90\, \text{kg} \]

Copyright © 2012 Pearson Education Inc.
Solving rigid-body equilibrium problems

- Read Problem-Solving Strategy 11.1.
- Follow Example 11.2 using Figure 11.8 below.
Will the ladder slip?

- Follow Example 11.3 using Figure 11.9 below.
Equilibrium and pumping iron

• Follow Example 11.4 using Figure 11.10 below.

(a) Tendon actually inserts close to elbow—moved here for clarity

(b) Body in equilibrium (dumbbell plus forearm)

We don’t know the sign of this component; we draw it positive for convenience.
Strain, stress, and elastic moduli

- Stretching, squeezing, and twisting a real body causes it to deform, as shown in Figure 11.12 below. We shall study the relationship between forces and the deformations they cause.

- Stress is the force per unit area and strain is the fractional deformation due to the stress. Elastic modulus is stress divided by strain.

- The proportionality of stress and strain is called Hooke’s law.
Tensile and compressive stress and strain

- **Tensile stress** = $F_\perp / A$ and **tensile strain** = $\Delta l / l_0$. **Compressive stress** and **compressive strain** are defined in a similar way. (See Figures 11.13 and 11.14 below.)

- **Young’s modulus** is tensile stress divided by tensile strain, and is given by $Y = (F_\perp / A)(l_0 / \Delta l)$. 

\[
\text{Tensile stress} = \frac{F_\perp}{A} \quad \text{Tensile strain} = \frac{\Delta l}{l_0}
\]

\[
\text{Compressive stress} = \frac{F_\perp}{A} \quad \text{Compressive strain} = \frac{\Delta l}{l_0}
\]
### Some values of elastic moduli

#### Table 11.1  Approximate Elastic Moduli

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s Modulus, $Y$ (Pa)</th>
<th>Bulk Modulus, $B$ (Pa)</th>
<th>Shear Modulus, $S$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>$7.0 \times 10^{10}$</td>
<td>$7.5 \times 10^{10}$</td>
<td>$2.5 \times 10^{10}$</td>
</tr>
<tr>
<td>Brass</td>
<td>$9.0 \times 10^{10}$</td>
<td>$6.0 \times 10^{10}$</td>
<td>$3.5 \times 10^{10}$</td>
</tr>
<tr>
<td>Copper</td>
<td>$11 \times 10^{10}$</td>
<td>$14 \times 10^{10}$</td>
<td>$4.4 \times 10^{10}$</td>
</tr>
<tr>
<td>Crown glass</td>
<td>$6.0 \times 10^{10}$</td>
<td>$5.0 \times 10^{10}$</td>
<td>$2.5 \times 10^{10}$</td>
</tr>
<tr>
<td>Iron</td>
<td>$21 \times 10^{10}$</td>
<td>$16 \times 10^{10}$</td>
<td>$7.7 \times 10^{10}$</td>
</tr>
<tr>
<td>Lead</td>
<td>$1.6 \times 10^{10}$</td>
<td>$4.1 \times 10^{10}$</td>
<td>$0.6 \times 10^{10}$</td>
</tr>
<tr>
<td>Nickel</td>
<td>$21 \times 10^{10}$</td>
<td>$17 \times 10^{10}$</td>
<td>$7.8 \times 10^{10}$</td>
</tr>
<tr>
<td>Steel</td>
<td>$20 \times 10^{10}$</td>
<td>$16 \times 10^{10}$</td>
<td>$7.5 \times 10^{10}$</td>
</tr>
</tbody>
</table>
Tensile stress and strain

- In many cases, a body can experience both tensile and compressive stress at the same time, as shown in Figure 11.15 below.

- Follow Example 11.5.

(a) Top of beam is under compression. Beam’s centerline is under neither tension nor compression. Bottom of beam is under tension.

(b) The top and bottom of an I-beam are broad to minimize the compressive and tensile stresses. The beam can be narrow near its centerline, which is under neither compression nor tension.
Bulk stress and strain

- Pressure in a fluid is force per unit area: \( p = \frac{F}{A} \).

- **Bulk stress** is pressure change \( \Delta p \) and **bulk strain** is fractional volume change \( \Delta V/V_0 \). (See Figure 11.16.)

- **Bulk modulus** is bulk stress divided by bulk strain and is given by \( B = \frac{-\Delta p}{(\Delta V/V_0)} \).

- Follow Example 11.6.
Sheer stress and strain

- **Sheer stress** is $F_{\parallel}/A$ and **sheer strain** is $x/h$, as shown in Figure 11.17.

- **Sheer modulus** is sheer stress divided by sheer strain, and is given by $S = (F_{\parallel}/A)(h/x)$.

- Follow Example 11.7.

Shear stress $= \frac{F_{\parallel}}{A}$  Shear strain $= \frac{x}{h}$
Elasticity and plasticity

- Hooke’s law applies up to point $a$ in Figure 11.18 below.
- Table 11.3 shows some approximate breaking stresses.

### Table 11.3 Approximate Breaking Stresses

<table>
<thead>
<tr>
<th>Material</th>
<th>Breaking Stress (Pa or N/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>$2.2 \times 10^8$</td>
</tr>
<tr>
<td>Brass</td>
<td>$4.7 \times 10^8$</td>
</tr>
<tr>
<td>Glass</td>
<td>$10 \times 10^8$</td>
</tr>
<tr>
<td>Iron</td>
<td>$3.0 \times 10^8$</td>
</tr>
<tr>
<td>Phosphor bronze</td>
<td>$5.6 \times 10^8$</td>
</tr>
<tr>
<td>Steel</td>
<td>$5 - 20 \times 10^8$</td>
</tr>
</tbody>
</table>