Chapter 14

Periodic Motion

PowerPoint® Lectures for
University Physics, Thirteenth Edition
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Goals for Chapter 14

• To describe oscillations in terms of amplitude, period, frequency and angular frequency
• To do calculations with simple harmonic motion
• To analyze simple harmonic motion using energy
• To apply the ideas of simple harmonic motion to different physical situations
• To analyze the motion of a simple pendulum
• To examine the characteristics of a physical pendulum
• To explore how oscillations die out
• To learn how a driving force can cause resonance
Introduction

- Why do dogs walk faster than humans? Does it have anything to do with the characteristics of their legs?

- Many kinds of motion (such as a pendulum, musical vibrations, and pistons in car engines) repeat themselves. We call such behavior *periodic motion* or *oscillation*.
What causes periodic motion?

- If a body attached to a spring is displaced from its equilibrium position, the spring exerts a **restoring force** on it, which tends to restore the object to the equilibrium position. This force causes oscillation of the system, or **periodic motion**.

- Figure 14.2 at the right illustrates the restoring force $F_x$. 
Characteristics of periodic motion

- The *amplitude*, $A$, is the maximum magnitude of displacement from equilibrium.
- The *period*, $T$, is the time for one cycle.
- The *frequency*, $f$, is the number of cycles per unit time.
- The *angular frequency*, $\omega$, is $2\pi$ times the frequency: $\omega = 2\pi f$.
- The frequency and period are reciprocals of each other: $f = 1/T$ and $T = 1/f$.
- Follow Example 14.1.
Simple harmonic motion (SHM)

- When the restoring force is directly proportional to the displacement from equilibrium, the resulting motion is called simple harmonic motion (SHM).

- An ideal spring obeys Hooke’s law, so the restoring force is $F_x = -kx$, which results in simple harmonic motion.
Simple harmonic motion viewed as a projection

- Simple harmonic motion is the projection of uniform circular motion onto a diameter, as illustrated in Figure 14.5 below.
Characteristics of SHM

- For a body vibrating by an ideal spring:

\[
\omega = \sqrt{\frac{k}{m}} \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}
\]

- Follow Example 14.2 and Figure 14.8 below.

(a) [Diagram of a spring with a mass attached, showing a force of 6.0 N and positions x = 0 and x = 0.030 m]

(b) [Diagram of a spring with a mass of 0.50 kg, showing positions x = 0 and x = 0.020 m]
The displacement as a function of time for SHM with phase angle $\phi$ is $x = A\cos(\omega t + \phi)$. (See Figure 14.9 at the right.)

Changing $m$, $A$, or $k$ changes the graph of $x$ versus $t$, as shown below.

(a) Increasing $m$; same $A$ and $k$
Mass $m$ increases from curve 1 to 2 to 3. Increasing $m$ alone increases the period.

(b) Increasing $k$; same $A$ and $m$
Force constant $k$ increases from curve 1 to 2 to 3. Increasing $k$ alone decreases the period.

(c) Increasing $A$; same $k$ and $m$
Amplitude $A$ increases from curve 1 to 2 to 3. Changing $A$ alone has no effect on the period.
Graphs of displacement, velocity, and acceleration

- The graph below shows the effect of different phase angles.

These three curves show SHM with the same period $T$ and amplitude $A$ but with different phase angles $\phi$.

- The graphs below show $x$, $v_x$, and $a_x$ for $\phi = \pi/3$.

  (a) Displacement $x$ as a function of time $t$

  $x(t) = A \cos(\omega t + \phi)$

  $x_{\text{max}} = A$

  $x_{\text{min}} = -A$

(b) Velocity $v_x$ as a function of time $t$

  $v_x(t) = -\omega A \sin(\omega t + \phi)$

  $v_{\text{max}} = \omega A$

  $v_{\text{min}} = -\omega A$

The $v_x$-$t$ graph is shifted by $\frac{1}{4}$ cycle from the $x$-$t$ graph.

(c) Acceleration $a_x$ as a function of time $t$

  $a_x(t) = -\omega^2 A \cos(\omega t + \phi)$

  $a_{\text{max}} = \omega^2 A$

  $a_{\text{min}} = -\omega^2 A$

The $a_x$-$t$ graph is shifted by $\frac{1}{4}$ cycle from the $v_x$-$t$ graph and by $\frac{1}{2}$ cycle from the $x$-$t$ graph.
Behavior of $v_x$ and $a_x$ during one cycle

- Figure 14.13 at the right shows how $v_x$ and $a_x$ vary during one cycle.

- Refer to Problem-Solving Strategy 14.1.

- Follow Example 14.3.
Energy in SHM

- The total mechanical energy \( E = K + U \) is conserved in SHM:

\[
E = \frac{1}{2} mv_x^2 + \frac{1}{2} kx^2 = \frac{1}{2} kA^2 = \text{constant}
\]
Energy diagrams for SHM

- Figure 14.15 below shows energy diagrams for SHM.
- Refer to Problem-Solving Strategy 14.2.
- Follow Example 14.4.

(a) The potential energy $U$ and total mechanical energy $E$ for a body in SHM as a function of displacement $x$

(b) The same graph as in (a), showing kinetic energy $K$ as well

At $x = \pm A$ the energy is all potential; the kinetic energy is zero.

At $x = 0$ the energy is all kinetic; the potential energy is zero.

At these points the energy is half kinetic and half potential.
Energy and momentum in SHM

- Follow Example 14.5 using Figure 14.16.
Vertical SHM

• If a body oscillates vertically from a spring, the restoring force has magnitude $kx$. Therefore the vertical motion is SHM.

• Follow Example 14.6.
Angular SHM

- A coil spring (see Figure 14.19 below) exerts a restoring torque $\tau_z = -\kappa \theta$, where $\kappa$ is called the torsion constant of the spring.

- The result is angular simple harmonic motion.

![Diagram of angular simple harmonic motion with a balance wheel and a spring. The spring torque $\tau_z$ opposes the angular displacement $\theta$.](image-url)
Vibrations of molecules

- Figure 14.20 shows two atoms having centers a distance $r$ apart, with the equilibrium point at $r = R_0$.

- If they are displaced a small distance $x$ from equilibrium, the restoring force is $F_r = -(72U_0/R_0^2)x$, so $k = 72U_0/R_0^2$ and the motion is SHM.

- Follow Example 14.7.
The simple pendulum

- A *simple pendulum* consists of a point mass (the bob) suspended by a massless, unstretchable string.

- If the pendulum swings with a small amplitude $\theta$ with the vertical, its motion is simple harmonic. (See Figure 14.21 at the right.)

- Follow Example 14.8.
The physical pendulum

- A physical pendulum is any real pendulum that uses an extended body instead of a point-mass bob.

- For small amplitudes, its motion is simple harmonic. (See Figure 14.23 at the right.)

- Follow Example 14.9.
Tyrannosaurus rex and the physical pendulum

- We can model the leg of *Tyrannosaurus rex* as a physical pendulum.

- Follow Example 14.10 using Figure 14.24 below.
Damped oscillations

- Real-world systems have some dissipative forces that decrease the amplitude.
- The decrease in amplitude is called *damping* and the motion is called *damped oscillation*.
- Figure 14.26 at the right illustrates an oscillator with a small amount of damping.
- The mechanical energy of a damped oscillator decreases continuously.

With stronger damping (larger $b$):
- The amplitude (shown by the dashed curves) decreases more rapidly.
- The period $T$ increases ($T_0 =$ period with zero damping).
Forced oscillations and resonance

- **A forced oscillation** occurs if a *driving force* acts on an oscillator.
- **Resonance** occurs if the frequency of the driving force is near the *natural frequency* of the system. (See Figure 14.28 below.)