Q1. (20 points) The graph below shows the velocity of an object moving on the x-axis as a function of time. At $t = 0$, the object is at position $x = 15 \text{m}$.

![Graph of velocity vs. time](image)

**a)** Where is the particle located at $t = 6 \text{s}$? (5pts.)

$\Delta x$ is the area under $(v_x \text{ vs. } t)$ curve between $t = 0$ and $t = 6 \text{s}$.

\[
\Delta x = \frac{1}{2} (8 \text{ m/s}) \times (4 \text{s}) + (8 \text{ m/s}) \times (2 \text{s}) = 32 \text{m}
\]

\[
\Delta x = x_6 - x_0 = 32 \text{m} = x_6 - 15 \text{m} \Rightarrow x_6 = 47 \text{m}
\]

**b)** What is the average velocity over the first 6 seconds period? (5pts.)

\[
\overline{v_x} = \frac{\Delta x}{\Delta t} = \frac{32 \text{m}}{6 \text{s}} = 5.33 \text{ m/s}
\]

**c)** Calculate the instantaneous acceleration at $t = 7 \text{s}$. (5pts.)

$a_x$ is the slope to the tangent to the $(v \text{ vs. } t)$ curve at $t = 7 \text{s}$:

\[
a_x = \frac{(2 \text{ m/s}) - (8 \text{ m/s})}{(10 \text{s}) - (6 \text{s})} = -1.5 \text{ m/s}^2
\]

**d)** Calculate the average acceleration between $t = 0 \text{s}$ and $t = 6 \text{s}$. (5pts.)

\[
\overline{a_x} = \frac{v_6 - v_0}{t_6 - t_0} = \frac{(8 \text{ m/s}) - (0)}{(6 \text{s}) - (0)} = 1.67 \text{ m/s}^2
\]
Q2. (20 points) A truck is driving down the road at a constant 22 m/s speed and it passes a car going only 11 m/s (call this t=0 and x=0). Although the car is slower, it also has a constant acceleration, so that 14 seconds later, the car passes the truck.

(a) (4 pts) How far down the road are they when the car passes the truck?

At \( t = 14 \text{ s} \), \( x_T = x_C \)

\[
x_T = V_T t = (22 \text{ m/s}) \times (14 \text{ s})
\]

\[
x_C = x_T = [308 \text{ m}]
\]

(b) (4 pts) What is the constant acceleration of the car?

Using \( \triangle x = V_0 t + \frac{1}{2} a t^2 \):

\[
x_C = 308 \text{ m} = (11 \text{ m/s}) \times (14 \text{ s}) + \frac{1}{2} a_C (14 \text{ s})^2
\]

\[
a_C = \boxed{1.57 \text{ m/s}^2}
\]

(c) (6 pts) At what time will the car and truck have the same speed?

When \( V_C = V_T = 22 \text{ m/s} \), the time can be found from \( V_C = V_0 + a_C t \).

\[
22 \text{ m/s} = 11 \text{ m/s} + 1.57 \text{ m/s}^2 \times t \implies t = \boxed{7 \text{ seconds}}
\]

(d) (3 pts) Where is the truck at that time?

At \( t = 7 \text{ s} \), the truck will be at \( x_T = V_T t \)

\[
V_T = (22 \text{ m/s}) \times (7 \text{ s}) = [154 \text{ m}]
\]

(e) (3 pts) Where is the car at that time?

At \( t = 7 \text{ s} \), the car will be at \( x_C = V_0 t + \frac{1}{2} a_C t^2 \)

\[
x_C = (11 \text{ m/s}) \times (7 \text{ s}) + \frac{1}{2} (1.57 \text{ m/s}^2) (7 \text{ s})^2
\]

\[
= [115.5 \text{ m}]
\]
Q3. (20 points) A stone is thrown from the edge of a building of height $h = 30\, \text{m}$ at an angle $\theta_0 = 37^\circ$ above the horizontal and with initial speed $v_0 = 15\, \text{m/s}$.

(a) Find the $x$ and the $y$ components of the initial velocity of the stone. (4 pts)

\[
V_{ox} = v_0 \cos \theta_0 = (15\, \text{m/s}) \cos 37^\circ
\]

\[
V_{ox} = \boxed{12\, \text{m/s}}
\]

\[
V_{oy} = v_0 \sin \theta_0 = (15\, \text{m/s}) \sin 37^\circ
\]

\[
V_{oy} = \boxed{9\, \text{m/s}}
\]

(b) Find the time it takes the stone to hit the ground? (6 pts)

\[
\Delta y = V_{oy} t - \frac{1}{2} g t^2
\]

\[
-30 = \left(9\, \text{m/s}\right) t - \frac{1}{2} \left(10\, \text{m/s}^2\right) t^2
\]

or:

\[
t^2 - (1.8)s t - (6s^2) = 0
\]

\[
t = \frac{(-1.8) \pm \sqrt{(-1.8)^2 + 4 \times 1 \times (-6s^2)}}{2}
\]

\[
= \frac{(1.8) \pm (5.2)}{2}
\]

\[
\Rightarrow t = 3.5\, \text{s} \quad \text{(the answer is rejected)}
\]

(c) At what distance from the base of the building will the object hit the ground? (2 pts)

\[
\Delta x = V_{ox} \Delta t = \left(12\, \text{m/s}\right) \times (3.5\, \text{s})
\]

\[
\Delta x = 42\, \text{meters}
\]

(d) What is the magnitude and direction of the velocity of the stone when it hits the ground? (Hint: this can be calculated without using your answer to part (c).) (8 pts)

\[
\vec{V} = \vec{V}_x + \vec{V}_y
\]

\[
\theta = \tan^{-1}\left(\frac{\vec{V}_y}{\vec{V}_x}\right)
\]

\[
V_x = V_{ox} = \boxed{12\, \text{m/s}}
\]

\[
V_y = V_{oy} - g \Delta t = (9\, \text{m/s}) - (10\, \text{m/s})^2 \times 3.5\, \text{s}
\]

\[
= -26\, \text{m/s}
\]

\[
V = \sqrt{(12\, \text{m/s})^2 + (-26\, \text{m/s})^2}
\]

\[
= 28.6\, \text{m/s}
\]

\[
\theta = \tan^{-1}\left(\frac{-26\, \text{m/s}}{12\, \text{m/s}}\right) = -65.2^\circ
\]
(18 pts) Projectile motion
You and a friend of yours with a not so good physics background throw an object between the top of two buildings. The buildings are 4m apart and have a difference of 3 m in their height, like in the picture. The objects are both thrown at the same speed of 5 m/s.

(a) (5 pts) Your friend thinks one needs to maximize the horizontal velocity and throws the object horizontally. Sadly the object doesn’t clear the gap. Prove this (after falling 3m, what is the horizontal displacement?)

\[ v_{ox} = 5 \text{ m/s} \quad \; v_{oy} = 0 \text{ m/s} \]
\[ x = x_0 + v_{ox} t \quad ; \quad x = 5 t \quad \; y = y_0 + v_{oy} t - \frac{1}{2} g t^2 \]
\[ y = -4.9 t^2 \]

After falling 3m: \[ -3 = -4.9 t^2 \implies t = 0.672 \text{ s} \]

the x - displacement \( x \) is: \[ x = 5 (0.672) = 3.36 \text{ m} < 4 \text{ m} \]

(b) (5 pts) How far below the top of the second building will the object hit (find the distance between the top edge of the second building and point P1)

When the object hits the building \( x = 4 \text{ m} \)

It takes an amount of time equal to:
\[ x = x_0 + v_{ox} t \quad ; \quad 4 = 5 t \quad ; \quad t = \frac{4}{5} = 0.8 \text{ s} . \]

In that time \( t \) falls
\[ y = -4.9 (0.8)^2 = -3.136 \text{ m} \]

0.136 m below the top

(c) (4 pts) When your turn comes, you throw the object at a 45° angle above the horizontal.

Your object does clear the gap. By how much? (find the distance between the edge of the second building and P2)

\[ v_{ox} = 5 \cos 45 = 3.53 \text{ m/s} \quad ; \quad v_{oy} = 5 \sin 45 = 3.53 \text{ m/s} \]
\[ y = y_0 + v_{oy} t - \frac{1}{2} g t^2 = 3.53 t - 4.9 t^2 \quad ; \quad \text{after falling 3m:} \]
\[ -3 = 3.53 t - 4.9 t^2 \implies 4.9 t^2 - 3.53 t + 3 = 0 \implies t = 0.22 \text{ sec} . \]

At that time the horizontal displacement is:
\[ x = (3.53) (0.22) = 0.78 \text{ m} . \]

It clears the gap by 0.31 m margin.

(d) (4 pts) Find the speed of your object as it hits the roof of the second building.

\[ v_x = v_{ox} = 3.53 \text{ m/s} \]
\[ v_y = -g t = 3.53 - 9.8 (1.22) = -8.43 \text{ m/s} \]

speed = \[ | \vec{v} | = \sqrt{(3.53)^2 + (-8.43)^2} = 9.13 \text{ m/s} \]