Physics 142 Midterm 2

UIC — Summer 2014

July 23, 2014

Last Name: 
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Signature: 

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1. You are allowed to use one 8.5" × 11" equation sheet as well as a calculator.

2. Textbooks, notes, as well as cell phones or any other forms of wireless communication are strictly prohibited in this or any exam. Giving or receiving aid in an examination is cause for dismissal from the University.

3. Perform the necessary calculations in the space provided — you will not receive credit for work done on scratch paper.

4. ALL WORK MUST BE SHOWN IN ORDER TO RECEIVE FULL CREDIT.

5. All answers should include appropriate units.

6. Clearly circle or box your answers.

Advice: Read each question completely before doing any part of it.
Possibly Useful Mathematical Expressions

Trig Identities:

\[ \sin^2 x + \cos^2 x = 1 \]

\[ \sin(-x) = -\sin x \quad \cos(-x) = \cos x \]

\[ \sin \left( \frac{\pi}{2} - x \right) = \cos x \quad \cos \left( \frac{\pi}{2} - x \right) = \sin x \]

\[ \sin(x + y) = \sin x \cos y + \cos x \sin y \]

\[ \cos(x + y) = \cos x \cos y - \sin x \sin y \]

Integrals:

\[ \int_a^b x^n dx = \frac{1}{n + 1} \left( a^{n+1} - b^{n+1} \right) \]

\[ \int_0^{2\pi} \sin^2(x) dx = \pi \quad \int_0^{2\pi} \cos^2(x) dx = \pi \]

\[ \int_a^b \frac{1}{x} dx = \ln \left( \frac{b}{a} \right) \]

\[ \int_0^{\pi/2} \frac{\sin \theta + \cos \theta - 1}{[(1 - \sin \theta)^2 + (1 - \cos \theta)^2]^{3/2}} d\theta \approx 3.20 \]

\[ \int_{\pi/2}^{\pi} \frac{\sin \theta - \cos \theta - 1}{[(1 - \sin \theta)^2 + (1 + \cos \theta)^2]^{3/2}} d\theta \approx 3.20 \]

\[ \int_0^{\pi} \frac{\sin \theta + \cos \theta + 1}{[(1 + \sin \theta)^2 + (1 + \cos \theta)^2]^{3/2}} d\theta \approx 3.20 \]
Multiple Choice Problems — 30 Points Total

Each problem is worth 3 points. Circle the correct answer(s). There is no partial credit for multiple choice questions.

1. Consider a constant voltage source with some number of resistors connected to it in parallel. As more resistors are added (still in parallel), the power supplied by the source
   (a) increases.
   (b) decreases.
   (c) does not change.

2. A light bulb is connected in the circuit shown in the figure with the switch $S$ open. All the wires have no resistance, the battery has no internal resistance, and the bulb is an ohmic resistor. When we close the switch, which statements below accurately describe the behavior of the circuit? (There may be more than one correct choice.)

   ![Circuit Diagram]

   (a) The brightness of the bulb will increase.
   (b) The brightness of the bulb will decrease.
   (c) The brightness of the bulb will not change.
   (d) The potential drop across $R_2$ will decrease.
   (e) The potential drop across $R_2$ will not change.

3. An electron, moving toward the west, enters a uniform magnetic field. Because of this field the electron curves upward (not towards the north — $up$). The direction of the magnetic field is
   (a) towards the north.
   (b) towards the south.
   (c) towards the west.
   (d) upward.
   (e) downward.
4. A charge is accelerated from rest through a potential difference \( V \) and then enters a constant magnetic field oriented perpendicular to its path. The field deflects the particle into a circular arc of radius \( R \). If the accelerating potential is increased to 4 \( V \), what will the new radius of the circular arc be?

(a) \( 4R \)  
(b) \( 2R \)  
(c) \( R \)  
(d) \( R/2 \)  
(e) \( R/4 \)

\[
\frac{\beta}{c} = \frac{mV}{qB} \quad \frac{1}{2}mv^2 = qV \quad v = \frac{qV}{\frac{q}{m}}
\]

5. The figure shows three long, parallel current-carrying wires. The magnitudes of the currents are equal and their directions are indicated in the figure. Which of the arrows drawn near the wire carrying current 1 correctly indicates the direction of the magnetic force acting on that wire?

(a) A  
(b) B  
(c) C  
(d) D  
(e) The net magnetic force on the current \( I_1 \) is equal to zero.

6. Consider a solenoid of length \( L \), \( N \) windings, and radius \( b \) (where \( L \) is much longer than \( b \)). A current \( I \) is flowing through the wire. If the length of the solenoid became twice as long (2\( L \)), and all other quantities listed above remained the same, the magnetic field inside the solenoid would

(a) remain the same.  
(b) become twice as strong.  
(c) become one half as strong.

\[
\beta = \mu_0 n I = \frac{\mu_0 N I}{L}
\]
7. In the figure, a straight wire carries a current $I$. The wire passes through the center of a toroidal coil. If the current in the wire is quickly reduced to zero, the induced current through the resistor $R$ is

(a) from $a$ to $b$.
(b) from $b$ to $a$.
(c) There is no induced current through the resistor.
(d) The answer depends on how the rate at which the current is decreasing changes with time (i.e. on the second derivative of the current).

8. A capacitor is charging in a simple R-C circuit with a DC battery. Which one of the following statements about this capacitor is accurate?

(a) There is a magnetic field between the capacitor plates because charge travels between the plates by jumping from one plate to the other.
(b) There is no magnetic field between the capacitor plates because no charge travels between the plates.
(c) There is a magnetic field between the capacitor plates, even though no charge travels between them, because the magnetic flux between the plates is changing.
(d) There is a magnetic field between the capacitor plates, even though no charge travels between them, because the electric flux between the plates is changing.

9. A resistor and an ideal inductor are connected in series to an ideal battery having a constant terminal voltage $V_0$. At the moment contact is made with the battery, which of the following statements is correct?

(a) The voltage across the resistor is $V_0$ and the voltage across the inductor is also $V_0$.
(b) The voltage across the resistor is $V_0$ and the voltage across the inductor is 0.
(c) The voltage across the resistor is 0 and the voltage across the inductor is $V_0$.
(d) The voltage across the resistor is 0 and the voltage across the inductor is also 0.
(e) The voltage across the resistor is $V_0/2$ and the voltage across the inductor is $V_0/2$.

10. A charged particle in a magnetic field can never experience an acceleration due to the magnetic force which acts upon it.

(a) True
(b) False
P1: Circuit Analysis—20 Points Total

Show your work for partial credit.

A battery provides EMF $\mathcal{E}$, has internal resistance $r$, and is connected with two resistors as well as two capacitors as shown in the figure. The capacitors are initially uncharged, the switch $S$ begins open, and $R = 5r$. $i_0$, $i_1$, and $i_2$ denote the currents through the relevant parts of the circuit (although the directions may not necessarily be correct). Express all your answers in terms of $r$, $C$, and $\mathcal{E}$.

a) At time $t = 0$ the switch $S$ is closed. What are the potential differences $V_{ac}$ and $V_{bc}$? (2 pts)

$$V_{bc} = 0$$

$$V_{ac} = 0$$

b) After infinite time has elapsed (and with the switch $S$ remaining closed) what are $V_{ac}$ and $V_{bc}$? (4 pts)

$$V_{ac} = i_1 R = \frac{3}{16} \mathcal{E}$$

$$V_{bc} = \frac{3}{8} \mathcal{E}$$

c) Write down a set of independent equations that will yield the solutions for the currents $i_0$, $i_1$, and $i_2$ flowing in the three branches of the circuit. Do NOT solve them. This answer may also include $q$ — the charge accumulated on the upper portion of the top capacitor. (6 pts)

$$i_0 = i_1 + i_2$$

$$\mathcal{E} - i_0 r - i_1 5r - i_1 10r = 0 = \mathcal{E} - i_0 r - i_1 15r$$

$$\mathcal{E} - i_0 r - \frac{q}{2C} - \frac{q}{C} = 0 = \mathcal{E} - i_0 r - \frac{3q}{2C}$$
For the remainder of this problem, we short-circuit points a and b of the circuit by connecting them with a resistanceless conducting wire.

d) Will there be any current flowing through the new wire immediately after it is added, and if so in what direction? (2 pts)

Yes - to the left since $V_{bc} > V_{ac} \Rightarrow V_b > V_a$.

e) What will the final (i.e. steady state) values for $V_{ac}$ and $V_{bc}$ be? (3 pts)

Capacitors still act like switches so

$$V_{ac} = V_{bc} = \frac{5}{16} \varepsilon$$

f) Starting from the time when the wire was added, what total charge flows through the short-circuiting wire? (3 pts) Hint: Think about conservation of charge.

Capacitors no longer are in series, so charge on top and bottom capacitors are different.

$$Q_{bp} = 2C V_{bp} = C V_{bc} = \frac{5}{16} \varepsilon$$

$$Q_{bottom} = C V_{bc} = \frac{5}{16} \varepsilon$$

$$= 2 \varepsilon$$

Recall (part b) $Q = C \varepsilon / 16$ (eq ii initial charge on capacitors) before wire added.

$$Q_{bottom} = -\frac{3}{2} Q$$

So $\frac{3}{2} Q$ must have flown through the wire.

But so $\frac{3}{2} \times 10/16 = \frac{15}{16} C \varepsilon$.

TOTAL (of 20)
A coaxial cable consists of a cylindrical conductor (i.e. a wire) of radius \(a\), surrounded by empty space (out to radius \(b\)), which is then surrounded by another cylindrical conductor (outer radius \(c\), as shown in the figure to the left. Both the inner and outer conductors of the cable carry a time-varying current of magnitude \(i = I_0 \sin(\omega t)\) (where \(\omega\) is some angular frequency) but the currents are in opposite directions. Assume these currents travel \textit{entirely} on the outer surfaces of the conductors. Let \(r\) be the distance from the axis of the cable to a given point \(P\), and let \(\hat{r}\) be the direction pointing radially outward from the axis of the cable. Express all your answers in terms of \(a, b, c, I_0, \omega, t\) and \(r\).

a) Assuming that \(\omega\) is small enough that one can ignore any displacement current, what is the magnitude of the magnetic field as a function of time for any point \(P\) with \(c < r\)? (2 pts)

\[
\begin{align*}
\mathbf{B} & = 0 \\
\text{and by symmetry}
\end{align*}
\]

b) Again, assuming \(\omega\) small, what is the magnitude of the magnetic field as a function of time for any point \(P\) with \(r < a\)? (2 pts)

\[
\begin{align*}
\mathbf{B} & = 0 \\
\text{and by symmetry}
\end{align*}
\]

c) Again, assuming \(\omega\) small, what is the magnitude of the magnetic field as a function of time for any point \(P\) with \(a < r < b\)? (6 pts)

\[
\mathbf{B} = \frac{\mu_0}{2\pi r} \int_{\mathbb{S}} \mathbf{J} \cdot d\mathbf{A} = \frac{\mu_0}{2\pi r} \left( I_0 \sin(\omega t) \right)
\]

\[
\mathbf{B} = \frac{\mu_0}{2\pi r} \left( I_0 \sin(\omega t) \right)
\]

Front view
d) Using your results from the previous sections, calculate the electric field for any point $P$ with $a < r < b$. (5 pts) Hint: Consider the magnetic flux through a rectangular loop with one edge along the axis of the central conductor.

By Gauss's law, radial component of $E$-field is 0.

By cylindrical symmetry, remaining $E$-field must be colinear w/ axis of cable.

By symmetry, $E$ must be zero along central axis.

$$\oint E \cdot dl = -\frac{d\Phi_B}{dt}$$

$$\Theta_{\Pi} = \int_0^b \Phi_B \cdot dA = L \int_0^b dB dr$$

$$\int E dy = EL$$

$$E = \frac{L}{2\pi} \sin \left(\frac{\pi}{2} \right) \ln \left(\frac{c}{a}\right)$$

\[ E = \frac{I_0}{2\pi} \sin (wt) \mu_0 \ln \left(\frac{c}{a}\right) \]

\[ L = \frac{\mu_0}{2\pi} \ln \left(\frac{c}{a}\right) \cdot 40m = \frac{\mu_0}{2\pi} \ln (2) \cdot 40m = 5.5 \times 10^{-6} \text{H} \]

\[ I = 700 \text{mA} \sin \left(\frac{377}{2} \cdot 12 \text{ms}\right) \]

\[ U = \frac{1}{2} L i^2 \]

\[ i = 1.3 \mu A \]

\[ c = 1.69 \mu A \]

\[ = 1.69 \mu A \]

\[ \text{TOTAL (of 20)} \]

9
A long wire of negligible thickness extends straight out to infinity in both the $+x$ and $+y$ directions, with a perfectly circular bend (of radius $R = 5.0$ cm) near the origin $O$. A constant current $I = 400$ mA passes through the wire. Point $P$ lies on the opposite side of the circular bend from the origin, in line with the infinitely straight segments of wire as shown (with position vector $-RI - RJ$), while point $a$ has position vector $-RI$ and point $b$ has position vector $-RI$.

a) What is the direction of the magnetic field at the point $P$? (2 pts)

Out of the page

b) What is the magnitude of the magnetic field at the point $P$ due to the long straight segment of wire extending from the point $a$ to infinity in the $+x$ direction? (3 pts)

\[ \oint d\ell \times \hat{r} = 0 \]

c) What is the magnitude of the magnetic field at the point $P$ due to the long straight segment of wire extending from the point $b$ to infinity in the $+y$ direction? (3 pts)

\[ \oint d\ell \times \hat{r} = 0 \]
d) What is the magnitude of the magnetic field at the point $P$ due to the circular curved segment of wire extending from the point $a$ to the point $b$? (7 pts)

We need to use Biot–Savart along a parameterized path. Take $\theta$ to be angle from line extending from origin to $a$.

\[
\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{d}\vec{r} \times \vec{r}}{r^2} = \frac{\mu_0 I}{4\pi} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{d\vec{r} \times \vec{r}}{r^2} \, d\theta
\]

\[
\vec{r}_p = \vec{r}_w + \vec{r}
\]

\[
\vec{r}_w = R\left(-\sin \theta \hat{i} + \cos \theta \hat{j}\right)
\]

\[
\frac{d\vec{r}_w}{d\theta} = R\left(-\cos \theta \hat{i} + \sin \theta \hat{j}\right)
\]

\[
- \vec{r}_w + \vec{r}_p = R\left(-\sin \theta \hat{i} - \cos \theta \hat{j}\right) + \vec{r} = -R \left[\left(1 + \sin \theta\right) \hat{i} + \left(1 - \cos \theta\right) \hat{j}\right]
\]

\[
r = R \sqrt{(1+\sin \theta)^2 + (1-\cos \theta)^2}
\]

\[
\frac{d\vec{r}_w}{d\theta} \times \vec{r}_p = R\left(-\cos \theta \hat{i} + \sin \theta \hat{j}\right) \times -R \left[\left(1 - \sin \theta\right) \hat{i} + \left(1 - \cos \theta\right) \hat{j}\right]
\]

\[
= -R^2 \left[-\cos \theta \left(1 - \cos \theta\right) \hat{i} + \sin \theta \left(1 - \sin \theta\right) \hat{j}\right]
\]

\[
= R^2 \left[\cos \theta - \cos^2 \theta + \sin \theta - \sin^2 \theta\right] \hat{k}
\]

\[
= R^2 \left[\cos \theta + \sin \theta - 1\right] \hat{k}
\]

\[
B = \frac{\mu_0 I}{4\pi} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{R^2 \left(\cos \theta + \sin \theta - 1\right)}{R^3 \left[(1 - \cos \theta)^2 + (1 - \sin \theta)^2\right]^{3/2}} \, d\theta
\]

\[
= 2.6 \mu T.
\]
P4: Simple Circuits — 15 Points Total

Show your work for partial credit.

First consider the following circuit which consists of a 3.4μF capacitor and a coil with a self-inductance of 0.080 H and no appreciable resistance. At \( t = 0 \) the capacitor has a charge of 5.4μC and the current in the inductor is zero.

a) What is the first time that the current will be at a maximum in this circuit? (4 pts)

\[
i(t) = i_0 \cos(wt + \phi) \\
Q = Q_0 \cos(wt + \phi) \\
\frac{dQ}{dt} = -Q_0 w \sin(wt) \\
\omega = \frac{1}{\sqrt{LC}} \\
\text{Max current is for } wt = \frac{\pi}{2} \\
t = \frac{\pi}{2} \sqrt{LC} = 8.19 \ \mu s
\]

b) What will the value of the current be at this time? (4 pts)

\[
|I_0| = Q_0 \omega = \frac{Q_0}{\sqrt{LC}} = 10 \text{ mA}
\]

Now, consider a different circuit, which consists of an ideal 60 V battery, a 42 H inductor having no resistance, a 24 Ω resistor, and a switch, all in series. Initially, the switch has been open for a very long time, and then at time \( t = 0 \) the switch is suddenly closed.

c) At what time will the potential difference across the inductor be 24 V? (4 pts)

\[
\frac{24 \text{ V}}{60 \text{ V}} = e^{-\frac{R}{L} t} \\
\ln \left( \frac{24}{60} \right) = -\frac{R}{L} t \\
t = \frac{L}{R} \ln \left( \frac{60}{24} \right) = 1.65 \ \text{s}
\]

d) What is the initial energy stored in the inductor? (3 pts)

\[
(\text{no current})
\]

TOTAL (of 15)