PRACTICE FINAL 1
Solutions
1) An ac source of period \( T \) and maximum voltage \( V \) is connected to a single unknown ideal element that is either a resistor, and inductor, or a capacitor. At time \( t = 0 \) the voltage is zero. At time \( t = T/4 \) the current in the unknown element is equal to zero, and at time \( t = T/2 \) the current is \( I = -I_{\text{max}} \), where \( I_{\text{max}} \) is the current amplitude. What is the unknown element?
   A) a resistor
   B) an inductor
   C) a capacitor
   D) an inductor or a capacitor
   Answer: C

2) When an \( LRC \) series circuit is driven at resonance, which of the following statements about the circuit are correct? (There may be more than one correct choice.)
   A) The impedance of the circuit is zero.
   B) The impedance of the circuit has its maximum value.
   C) The impedance of the circuit has its minimum value.
   D) The inductive reactance and the capacitive reactance are both zero.
   E) The inductive reactance and the capacitive reactance are exactly equal to each other.
   Answer: C, E

3) If the magnetic field of an electromagnetic wave is in the \( +x \)-direction and the electric field of the wave is in the \( +y \)-direction, the wave is traveling in the
   A) \( xy \)-plane.
   B) \( +z \)-direction.
   C) \( -z \)-direction.
   D) \( -x \)-direction.
   E) \( -y \)-direction.
   Answer: C

4) If an electromagnetic wave has components \( E_y = E_0 \sin(\kappa x - \omega t) \) and \( B_z = B_0 \sin(\kappa x - \omega t) \), in what direction is it traveling?
   A) \( -x \)
   B) \( +x \)
   C) \( +y \)
   D) \( -y \)
   E) \( +z \)
   Answer: B

5) Which one of the following is an accurate statement about light?
   A) When light strikes a surface at Brewster's angle, the reflected and transmitted light are both 100% polarized.
   B) When light strikes a surface at Brewster's angle, it is completely reflected at the surface.
   C) When light strikes a surface at Brewster's angle, only the reflected light is 100% polarized.
   D) When light strikes a surface at the critical angle, only the reflected light is 100% polarized.
   E) When light strikes a surface at the critical angle, all the light passes through the surface.
   Answer: C
6) In a single-slit diffraction experiment, the width of the slit through which light passes is reduced. What happens to the width of the central bright fringe?
   A) It stays the same.
   B) It becomes narrower.
   C) It becomes wider.
   D) Its behavior depends on the wavelength of the light.
   Answer: C

7) Light from a monochromatic source shines through a double slit onto a screen 5.00 m away. The slits are 0.180 mm apart. The dark bands on the screen are measured to be 1.70 cm apart. What is the wavelength of the incident light?
   A) 457 nm
   B) 306 nm
   C) 392 nm
   D) 612 nm
   E) 784 nm
   Answer: D

8) The critical angle in air for a particular type of glass is 39.0°. What is the speed of light in this class glass? \((c = 3.00 \times 10^8 \text{ m/s})\)
   A) \(1.97 \times 10^8 \text{ m/s}\)
   B) \(1.94 \times 10^8 \text{ m/s}\)
   C) \(1.91 \times 10^8 \text{ m/s}\)
   D) \(1.89 \times 10^8 \text{ m/s}\)
   E) \(2.00 \times 10^8 \text{ m/s}\)
   Answer: D

9) When light goes from one material into another material having a HIGHER index of refraction
   A) its speed, wavelength, and frequency all decrease.
   B) its speed and wavelength decrease, but its frequency stays the same.
   C) its speed decreases but its wavelength and frequency both increase.
   D) its speed decreases but its frequency and wavelength stay the same.
   E) its speed increases, its wavelength decreases, and its frequency stays the same.
   Answer: B

10) A single-slit diffraction pattern is formed on a distant screen. If the width of the single slit through which light passes is reduced, what happens to the width of the central bright fringe? Assume the angles involved remain small.
    A) The central bright fringe remains the same size.
    B) The central bright fringe becomes wider.
    C) The central bright fringe becomes narrower.
    D) The effect cannot be determined unless the distance between the slit and the screen is known.
    Answer: B
1 Propagating Light

Reflection and Refraction

In the figure, a laser positioned on a ship is used to communicate with a small two-man research submarine resting on the bottom of a lake. The laser is positioned 12 m above the surface of the water, and it strikes the water 20 m from the side of the ship. The water is 76 m deep and has an index of refraction of 1.33. How far horizontally is the submarine from the side of the ship?

Answer: 84.1 m

Polarization

The following are positioned in sequence: A source of natural light of intensity $I_0$; three ideal polarizers $A$, $B$, and $C$; and an observer. Polarizer axis angles are measured clockwise from the vertical, from the perspective of the observer. The axis angle of polarizer $A$ is set at 0° (vertical), and the axis angle of polarizer $C$ is set at 50°. Polarizer $B$ is set so that the beam intensity is zero at the observer. Give two angles (between 0° and 180°) which are possible axis angle settings of polarizer $B$?

Answer: 90° and 140°
Problem 10
(a) Poynting flux: \( S = 10^3 \) W/m\(^2\). Area: \( A = 10^{-4} \) m\(^2\). Time interval: \( \Delta t = 30\) s.

Energy absorbed = \( SA \Delta t = (10^3)(10^{-4})(30) = 3 \) J .

(b) Radiation pressure: \( P = S/c \) for the case of total absorption. Force on surface:

\[
F = PA = SA/c = \frac{(10^3)(10^{-4})}{(3 \times 10^8)} \approx 3.3 \times 10^{-10} \) N .

(c) 100% Reflection \( \Rightarrow \) no energy is absorbed.

Problem 3 Electromagnetic Wave.

(a) \( \vec{B} = B_0 \cos(wt + Ky) \hat{z} \)

\( \vec{E} = E_0 \cos(wt + Ky) \hat{y} \)

(b) see adjacent figure

(c) \( S = \frac{1}{4\pi} \int \vec{E} \cdot \vec{d}a = \frac{1}{4\pi} B_0^2 \cos^2(wt + Ky)(-y) \).

\[
\int \vec{E} \cdot \vec{d}a = \frac{AcB^2}{4\pi} \cos^2(wt + Ky).
\]

Instantaneous

\[
U = \frac{1}{2} \int P(t) dt = \frac{1}{4\pi} \int \left( \cos^2(wt + Ky) \right) dt.
\]

\[
\Rightarrow U = \frac{AcB^2}{4\pi} \cdot \frac{1}{20n} = \frac{5AB^2c}{2w}
\]

\[
[U] = [LJ^2 \cdot [UJ] \cdot [LJ^3 \cdot [LJ] = [U] \text{ TRUE!}
\]

\[
[C \cdot \frac{1}{[T]}]= \text{UNITS CONSISTENT!}
\]
Problem 11

(a) In the $\omega = 0$ steady state, all currents are constant in time. The self-inductor acts as a wire with zero resistance, shorting out the capacitor and ensuring $I_C = 0$ (there would be no current in this branch even if we replaced the capacitor with a resistor). Thus we effectively have a resistor $R$ in series with a zero-resistance wire, and $I_L = I_R = V_0/R$. (Notice, even though it is irrelevant here, that the reactance of the capacitor $(1/\omega C)$ is infinitely high.)

(b) With $\omega$ infinitely high, the reactance of the self-inductor $(\omega L)$ becomes infinitely large and thus $I_L = 0$. The reactance of the capacitor $(1/\omega C)$ goes to zero. We effectively have a resistor $R$ in series with a zero-reactance capacitor, and our peak current values are simply $I_C = I_R = V_0/R$.

(c) We would expect the peak value of $I_R$ to decrease from the value $V_0/R$ found in part (a) as we increase $\omega$ to non-zero values. The effective impedance of the parallel combination of the capacitor and the self-inductor was zero with $\omega = 0$. For $\omega \neq 0$ it can only increase, as the self-inductor will no longer act as zero-Ohm wire and the reactance of the capacitor $(1/\omega C)$ will also be non-zero. (This expectation can be confirmed by an exact calculation, if you are feeling ambitious.)

(d) The frequency is $\omega = 1/\sqrt{LC}$. (What else could it be?) This is the natural oscillation frequency of the $LC$ sub-circuit alone. We will have an oscillating current in the $LC$ sub-circuit that dissipates no energy. The voltage across the capacitor will be exactly offset by the power-supply voltage at all times, so that there will never be any voltage across the resistor. Hence, $I_R = 0$. (None of this is very intuitive.)
2 Interfering Radio Stations

Assume that Rupert Murdoch has unleashed a dastardly scheme to disturb WBEZ listeners by setting up a radio transmitter across the street from the WBEZ antenna at Navy Pier. While Rupert would love to broadcast Fox News Radio on the same frequency, he knows that this is illegal — WBEZ owns the rights to broadcast at 91.5 MHz in Chicago. However, Rupert knows enough physics to realize that if he broadcasts the same exact NPR feed as WBEZ, it will still cause interference. Assume that Rupert broadcasts the same signal as WBEZ in phase, but from an antenna a distance of $4\lambda$ from the WBEZ antenna (where $\lambda$ is the wavelength of the radio signal).

a) Radio waves travel at the speed of light — using this, determine the wavelength of the signal transmitted by WBEZ.

We can determine the wavelength from the equation $v = \lambda f$ where $v = c$ is the speed of light. This gives

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{91.5 \times 10^6} = 3.28 \text{ m}.$$  

b) Assume that a person is at point $P$ with his radio tuned to 91.5 MHz. Point $P$ is a distance $d$ from the WBEZ antenna, and the line from $P$ to the WBEZ antenna is at right angles to the line connecting the WBEZ antenna with Rupert Murdoch’s. Using this, determine the smallest $d > 0$ for which the signals interfere (completely) constructively.

The distance from Rupert’s antenna to $P$ is (by the Pythagorean theorem) $l = \sqrt{(4\lambda)^2 + d^2}$. The criteria for constructive interference is then $l - d = m\lambda$ for $m = 0, 1, 2, \ldots$. Solving for $d$ in terms of $m$ we obtain

$$\sqrt{16\lambda^2 + d^2} - d = m\lambda \implies 16\lambda^2 + d^2 = (m\lambda + d)^2 = \lambda^2 + m^2 \lambda^2 + 2m\lambda d \implies d = \frac{\lambda(16 - m^2)}{2m}.$$  

Since $d$ is a distance it must be positive, and so we must have $m < 4$. Plugging in for $m = 1, 2, 3$ we obtain the smallest $d$ for $m = 3$, in which case we have $d = 3.83 \text{ m}$.

c) Using the same information as above, determine the largest $d$ for which the signals interfere (completely) destructively.

The criteria for destructive interference is $l - d = (n + \frac{1}{2})\lambda$ for $n = 0, 1, 2, \ldots$. This is the same algebra as above replacing $m \to (n + \frac{1}{2})$, and so we have

$$d = \frac{\lambda(16 - (n + \frac{1}{2})^2)}{2(n + \frac{1}{2})}.$$  

We will clearly get the largest value for $d$ when $n + \frac{1}{2}$ is as small as possible (since we are subtracting it off and then dividing by it), and so this means we must have $n = 0$. Plugging this in, we get $d = 51.6 \text{ m}$. 